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IDENTIFICATION OF FINITE STATE MODELS OF A HUMAN OPERATOR

Richard A. Miller
Industrial and Systems Engineering

For the Period
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U.S. AIR FORCE
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This report describes the work completed under AFOSR Grant No. 77-3152. The abstract structure of the discrete control problem as it relates to manned systems has been refined and clarified. It has been shown that input-output data (information inputs to the operator, decision outputs from him) are representable by stochastic automata, a special type of discrete parameter, discrete state stochastic process. Further, the detailed structure of these systems has been examined and it has been shown that automata based on 1^{th} order state

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spaces (i.e., states are sequences of outputs including the present and preceding l-1 outputs) serve as excellent surogates for more general systems. These l^th order systems have the distinct advantage of being observable from input-output data, and transition probability estimates can be easily constructed. Identification and parameter estimation algorithms based on l^th order systems have been developed.

The AAA system simulated by AMRL has been studied in detail and the structure of a stochastic automaton representation has been constructed. This model was used to define explicit data requirements for discrete control modeling of this system and to design the discrete control experiments to be performed on it. Data obtained from these experiments have been analyzed and the results are presented.

The results are a set of state transition matrices and probability distributions which stochastically characterize, as a function of the input when the operator will switch the mode of operation or configuration of the system and what the new configuration will be. The results show that finite state models of the human operator performing discrete control tasks can be developed and identified from data. They further verify the feasibility of using a hierarchical structure to avoid combinatorial problems and maintain identifiability.

IDENTIFICATION OF FINITE STATE MODELS OF HUMAN OPERATORS

Report
on
AFOSR Grant # 77-3152
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December 1, 1977

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ABSTRACT

This report describes the work completed under AFOSR Grant No. 77-3152. The abstract structure of the discrete control problem as it relates to manned systems has been refined and clarified. It has been shown that input-output data (information inputs to the operator, decision outputs from him) are representable by stochastic automata, a special type of discrete parameter, discrete state stochastic process. Further, the detailed structure of these systems has been examined and it has been shown that automata based on l^{th} order state spaces (i.e., states are sequences of outputs including the present and preceding $l-1$ outputs) serve as excellent surogates for more general systems. These l^{th} order systems have the distinct advantage of being observable from input-output data, and transition probability estimates can be easily constructed. Identification and parameter estimation algorithms based on l^{th} order systems have been developed.

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new configuration will be. The results show that finite state models of the human operator performing discrete control tasks can be developed and identified from data. They further verify the feasibility of using a hierarchical structure to avoid combinatorial problems and maintain identifiability.

I. Introduction

This research is based on the hypothesis that many manual control systems are hierarchical in structure and that many upper level tasks are best described in discrete control terms. Loosely, a task is a discrete control task if an operator is attempting to coordinate or control the activities of some system with a finite number of control alternatives. Methods for modeling such systems are lacking. In particular, there is a need for mathematical structures which are rich enough in properties to provide insight into discrete system behavior and yet are simple enough to be useful for parameter estimation and analysis without an excessive amount of computation. The research completed to date with the support of AFOSR Grant #77-3152 has shown that stochastic automata are very viable structures for these purposes.

The major tasks which were to have been accomplished at this point of the research are the following:

1. Develop discrete control model structure.
2. Develop parameter estimation algorithms.
3. Write computer code for data analysis and parameter estimation.
4. Simulation experiments at AMRL.
5. Analysis of data.

Each of these tasks has been completed. The abstract structure of the discrete control problem has been defined and clarified. A detailed multilevel system description of the system simulated by AMRL has also been constructed. The

parameter estimation and data analysis methods have been developed and computer programs have been written and used to analyze the simulation data.

Detailed discussions of the research are given in the following sections of the report. Abstract set theoretic descriptions of discrete control systems and the general identification problem are provided in sections II and III. Constructive specification and more detailed models are discussed in sections IV and V. In section VI, the identification procedures specialized to the AMRL system are considered and a brief summary of the analysis procedures is given in section VII. A summary of the results of data analysis is presented in section VIII.

II. Abstract Structure of Discrete Control Systems

The block diagram shown in Figure 1 is an abstraction of the type of system of interest. The major components are the controlled system S, an interface system I, and the controller or decision maker DM. The system S is shown as consisting of several subsystems. These subsystems might be actual physically interconnected systems which operate simultaneously, or they might represent the dynamics of S under different modes of operation. The decision maker coordinates the operation of S (for example, switches the mode of operation) through use of the control m. m is assumed to have a finite number of possible values, one per

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*Tables and figures are in appendices at the end of the report.

alternative available to the decision maker. Any continuous control tasks also performed by the operator are assumed to be performed directly at the level S and not at the DM level.

The information interface system is also shown as consisting of multiple information sources and/or multiple information processing schemes. The decision maker exercises control over the operation of I via the control n. The interface system is considered to be some physical system such as a bank of instruments or a computer. Any information processing done by the operator himself is not represented by I, but included in DM.

The modeling objectives all deal with the DM component of this system, i.e., the human operator or decision maker. At this point only an input-output description is required. In terms of input-output behaviors, the system DM is abstractly equivalent to a relation

$$DM \subseteq Q \times (M \times N)$$

where the defining objects Q, M, N are sets of time functions, the input and output behaviors. Formally, if T denotes the time set of task then

$$Q \subseteq C^T; \quad M \subseteq D^T; \quad N \subseteq E^T$$

The superscript notation is used to identify sets of the following type

$$C^T = \{ q \mid q: T \rightarrow C \}$$

That is, C^T is the set of all functions on domain T into the set C. The remaining objects are defined similarly. The elements of the set C are distinct inputs to the operator. C may be multi-dimensional in the sense that it is a cartesian product on more elementary sets. The sets D and E are the control alternative sets

available to the operator. Elements in D are information system alternatives; E contains the alternatives by which the system S is controlled. In the problems of interest the decision maker has available only a finite number of alternates which means that the sets D and E are of finite cardinality.

The relation DM is viewed as the basic structure to be examined experimentally and described mathematically. The main steps in representing this structure are outlined below.

The basic system is the input-output relation

$$DM \subseteq Q \times (M \times N)$$

Every relation of this type can be decomposed into two systems, an input-state system and a state-output system. Specifically, there exist (non-unique) sets F and Z and systems S_1 and S_2 such that

$$Z \subseteq F^T$$

and

$$S_1 \subseteq Q \times Z$$

$$S_2 \subseteq Z \times (M \times N)$$

and

$$DM = S_2 \circ S_1.$$

The set Z is the set of state trajectories for DM and F is the state space. Essentially this construction says that underlying any input-output relation is a pair of relations, the input-state relation S_1 and the state-output relation S_2, whose composition is the relation DM itself. It is convenient to assume, with no loss of generality, that S_2 is a static system. That is, knowledge of the state at time t

is sufficient to determine the output at time t. More precisely S_2 static means there is a map h,

$$h: F \rightarrow D \times E,$$

such that for every element $(Z, (m, n)) \in S_2$ and for every $t \in T$,

$$(m(t), n(t)) = h(Z(t)).$$

The information about the system response is therefore encoded primarily in the input-state system S_1.

A general stochastic characterization of the DM system is most easily based on the system S_1. Recall that

$$S_1 \subseteq Q \times Z$$

A probability space can be defined with S_1 as the sample space, and the probability measure on DM follows through the static system S_2. The result is that DM can be considered to be the sample space of a probability space which is constructed via the state decomposition of DM. Stochastic processes appropriate for analysis and detailed modeling are now outlined.

III. A Stochastic Process Representation of the System

To each input, output and state is assigned a numerical name. Formally,

$$g_I : C \rightarrow N_I \quad ; \quad N_I = \{1, 2, 3, \cdots k_I\}$$

$$g_o : D \times E \rightarrow N_o \quad ; \quad N_o = \{1, 2, 3, \cdots k_o\}$$

$$g_s : F \rightarrow N_s \quad ; \quad N_F = \{1, 2, 3, \cdots k_s\}$$

A finite number of inputs are identified with g_I, numerical names are assigned to outputs via g_o and states are similarly identified with the map g_s.

A random variable defined on the sample space S_1 is a map of the form

$$x : S_1 \to N_I \times N_s$$

Now let X denote the set of all such random variables and let T' denote the sample point time set,

$$\mathbf{X} = \{x \mid x : S_1 \to N_I \times N_S\}$$

$$T' = \{0, \delta, 2\delta \cdots\}, \; T' \subset T$$

A stochastic process $\mathcal{S}$ defined on S_1 is a family of random variables indexed by time,

$$\mathcal{S} : T' \to X$$

For convenience a random variable in the range of $\mathcal{S}$, $\mathcal{S}(t) \in \mathbf{X}$, is denoted by the symbol x_t. The defining characteristic of the process is

$$x_t(q, z) = \{g_I\,(q(t)),\; g_s\,(z(t))\}, \quad (q, z) \in S_1$$

That is, the random variable x_t produces as values the numerical name of the input and state at time t. A sample path of the process corresponding to an outcome $(q, z) \in S_1$ is then a time function of the form

$$s(q, z) = \{(t, (n, m)) \mid (n, m) = x_t\,(q, z)\}$$

Clearly, points on the sample path are the random variable values at that point. The ensemble $\mathcal{E}$ corresponding to S_1 is then the set of all sample paths,

$$\mathcal{E} = \{s(q, z) \mid (q, z) \in S_1\}.$$

These structures are the basic objects of the required detailed mathematical models. Before developing detailed representations, the "event" characteristics of the sample paths are clarified.

The sets N_I, N_o, N_s are assumed to be finite for this problem. That is, there are only a finite number of inputs, states and outputs of interest. This suggests

that in the limiting case where the time between samples, δ, approaches zero the sample paths would be piecewise constant. That is, the input symbol applied remains the same for some period of time before it changes. Similarly, the state is constant for some period of time. An event can then be defined as either a change in input or a change in state. Furthermore any sample path is completely characterized by a sequence of pairs of the form

$$(i_1, t_1), (i_2, t_2) \cdots (i_n, t_n) \cdots$$

where the first symbol in each pair, say i_n, is the symbol applied and the second element of the pair, t_n, is the length of time that symbol is applied. Sample path behaviors are therefore characterized by the symbol applied and the length of time the symbol is applied.

IV Constructive Specification

The probabilistic apparatus considered to this point characterizes the overall sample space, i. e., the behavior of the system DM, but does not address the properties of any given sample path. Data analysis and system representation are greatly simplified by a representation in terms of a state transition function. Such a function would establish the probability of occupying any state at the next time point $t + \delta$ as a function of the state occupied at time t and the input in the interval between. In general, the transition also depends on the length of time the given state has been occupied.

A state occupancy probability distribution is a map of the form

$$\vartheta : N_s \rightarrow [0, 1]$$

which associates the probability of occupancy with each state in this state space.

If Θ denotes the set of all such probability distributions, the state transition map is a map of the form

$$\phi : T' \times N_s \times N_I \rightarrow \Theta$$

If $\theta\,(t + \delta)$ is the state occupancy distribution at time $t + \delta$, the input is symbol i and state j is currently occupied and has been for τ units,

$$\theta\,(t + \delta) = \phi\,(\tau, j, i).$$

In other words, the next distribution is conditioned on the input, the state and the time since the last state change event. A major part of the modeling exercise is to identify the map ϕ.

Before turning to detailed representation of the system, outputs must be reintroduced. It was assumed that states were defined in such a way that outputs were deterministically related to states, i. e.,

$$n : F \rightarrow D \times E .$$

Now, in terms of state names, consider a map h_s,

$$h_s : N_s \rightarrow N_O$$

defined so that

$$h_s \cdot g_s = g_O \cdot h.$$

In terms of a commutative diagram

$$\begin{tikzcd}
F \arrow[r, "h"] \arrow[d, "g_s"'] \arrow[dr, dashed] & D \times E \arrow[d, "g_O"] \\
N_s \arrow[r, "h_s"'] & N_O
\end{tikzcd}$$

The map h_s is defined so that the same output name is reached given the state

name as is reached from first assigning the output and thus identifying the output name. The map h_s induces a partitioning of the set N_s. For each element $k \in N_o$ there exist a subset of N_s, say $h_s|k$ such that for each element $j \in h_s|k$, $h_s(j) = k$. The probability of output k is therefore

$$\sum_{i \in h_s|k} \phi(i)$$

where ϕ is the appropriate state occupancy distribution.

The state transition map ϕ and output assignment map h_s can be used to establish the conditional probability of an output sample path given the input sample path and initial state occupancy distribution. Furthermore, matrix representations are possible.

The map ϕ assigns k_s probabilities to each of k_s states given the length of state occupancy t and the input i. Therefore, ϕ is represented by a family of $k_s \times k_s$ matrices each of which is parameterized by the input and time. Specifically let $P(i, \tau)$ denote the $k_s \times k_s$ matrix whose entry in row m column n is the probability that state n will be occupied at time $t + \delta$ given the state at t is m and m has been occupied for τ units, i. e., the m^{th} row of $P(i, \tau)$ is the state occupancy distribution at $t + \delta$ given the above mentioned information.

These structures can be simplified considerably if the state to which the next transition is made is independent of the time at which the transition occurs, assuming constant input. If s denotes the time between two state change events, t_1 is time of last event, the state at t_1 is i, the state at $t_1 + \delta$ is j, and the input throughout is k, this independence assumption requires

$$\text{Prob}\,(Z(t_1 + s) = j \quad Z(t_1) = i \text{ and } q(t_1) = k) =$$

$$P(k, s)_{i,j} = \hat{P}(k)_{i,j} * f(s)$$

$P(k, s)_{i,j}$ is the i, jth element of the transition matrix P(k, s), $\hat{P}(k)$ is the marginal transition matrix computed from P(k, t) by integrating out t; similarly $\tilde{P}(s)$ is the marginal obtained by integrating out the input dependence. $\hat{P}(k)$ then defines conditional state transition probabilities at the next event, f is the distribution of event occurence times in the presence of constant input k. The independence assumption greatly reduces the complexity of these stochastic automata models and similarly reduces data required for estimation and yet leaves a robust structure in the sense that it has the power to represent diverse sets of data.

The stochastic automata described above, i.e., a family of transition matrices P(k, t) and the output assignment map h_s are the basic components of any input-output representation of the human operator in a discrete control system. The identification problem is one of determining the state space and estimating the state transition probabilities from the existing data. The state space of the operator is never known beforehand and state sequences are therefore certainly not observed.* Recall that DM was a set of input time function-output time function pairs. The sample paths observed then do not contain the state trajectories and standard maximum likilihood estimates cannot be used to estimate the transition probabilities of a general stochastic automaton. An observable approximation based on lth order Markov processes is discussed in the next section.

*These states can be thought of as the operator's state of information and the outside observer does not know the operator's knowledge of the system he is controlling. Furthermore, he does not know how the operator represents that information.

V. An Observable Approximation

Let A_1 denote a general stochastic automaton of the type defined in the previous section. Let A_2 denote a second stochastic automaton constructed from a family of l^{th} order Markov chains. Assuming that there are k_o distinct decision alternatives, i. e., k_o elements in the set D x E, an l^{th} order Markov system defined on this output space has exactly l^{k_o} states. Each state is a sequence consisting of the current and the previous l-1 outputs. The output assignment map for this system is obvious. Note that the state sequence for system A_2 is observable from the input-output data. The l^{th} order Markov system will be used to approximate the more general, but unobservable, system A_1.

Suppose that both A_1 and A_2 are given and recall that the relation DM consists of pairs of input-output functions of the form (q, o).* Now, let $p_1(o|q)$ denote the probability that output sequence o is produced by system A_1 given the input sequence q. Let $p_2(o|q)$ denote the same for system A_2. The system A_2 is said to be input-output equivalent to A_1 if p_1 and p_2 are sufficiently close for all pairs (q, o) of interest.

It is shown in Appendix 1 that if time varying transition probabilities are allowed in the system A_2, then $p_2(o|q)$ can be made to exactly equal p_1. When the transition probabilities are constrained to be constants, the error in approximation is a function of the order of the chain l and the transition probabilities of the system A_1. Specifically, the proportional error in the input-output transition probabilities is bounded from above by

*o is used to represent the pair (m, n)

$$2(\lambda_{max})^{t-2}$$

where t is the time of the transition and λ_{max} is the maximum eigenvalue of $P_l(q(t))$ which is the transition matrix of Λ_l using the appropriate input q(t). The maximum eigenvalue is the largest eigenvalue less than one. Hence the bound decreases with time.

The largest error incurred is at t = 1, hence the order of the chain directly controls the error. By selecting l sufficiently large, the error will be small. A formal proof of the bound is found in Appendix 1. Intuitively, the bound is closely tied to the transient behavior of the system. With the l^{th} order structure, estimates of state transition probabilities cannot be made until at least l transitions have occurred. If the transient effects have decayed by this time, the approximation is quite good.

The l^{th} order Markov approximation therefore works well when l is sufficiently large. Obviously, the maximum eigenvalue cannot be known beforehand, but by successively testing the hypothesis that the system is order l against the alternate that it is order l+1, one can get a statistical determination of the chain order. Experience with simulated systems (in Appendix 2) has shown that such procedures work very well and the required order is substantially lower than the upper bound indicates. Essentially then, l^{th} order Markov processes conditioned by the input provide the statistical apparatus necessary to identify the state space of the finite state system and to estimate transition probabilities. The details of the parameter estimation procedures for such systems are presented in Appendix 3.

In summary, it has been argued that the data obtained from a discrete control

system is of the same abstract structure as a general stochastic automata. It has been further argued that stochastic automata based on 1^{th} order Markov systems are excellent surogates for the more general systems and they are observable from data. The application of these concepts to the AMRL simulator is discussed in the next section.

VI. Empirical Testing

Structure of the System

The basic macroscopic stucture of the simulated system is sketched in Figure 2. These diagrams identify all major system components and explicitly show the points at which discrete decisions made by the system operators influence the system configuration.

Rectangular blocks are used to define hardware and human components, circles with summation signs denote signal summing junctions, and circles with numbers denote off page signal connections. The symbol is used to denote logical variables or switches controlled by the operators. These blocks denote the outputs of the discrete decision systems which are the primary subject of the analysis.

Figure 2 together with the state definitions which are provided below, can be used to establish the system configuration during any operating mode and any operator activity. Table 1 is a list of all such indicator variables and the enabling conditions for each.

Model Structure

The modeling and analysis effort will be based on a decomposition of the

system in terms of a basic sequence of activities or functions. For each activity, the system configuration decisions will be further broken down and described in terms of a commander subsystem, an angle operator subsystem, and hardware subsystems. All identified system components and functions are listed in Table 2. Also included in the same table is a list of major states for each component subsystem and the operator responsible for controlling the subsystem state.

Time Sequence or Activity States

The major functions performed by the system are search, acquire, track and fire. In any given operational situation the system will pass through all or a subset of these phases and transitions are indicated by a fairly well defined set of exogenous events. The state transition diagram together with enabling inputs are shown in Figure 3. The search state is used when an acceptable target has not been detected or acquired. Acquisition follows and is noted by a switch to a manual tracking mode. Tracking starts when the angle operator has minimized errors. Tracking continues in either an automatic or manual mode. The fire mode is entered when the data ready event occurs or the computer is shunted.

Commander--Angle Operator States

Separate angle operator, commander and hardware configuration systems are defined for each activity state. This results in a total of eight distinct systems plus hardware systems. The state spaces for all such systems are defined via state trees presented in Figures 4-11.

For purposes of analysis, the activity state will serve as the major indicator and the angle operator states and commander states will establish current macroscopic activity and configuration variables. Hardware states will be considered

outputs (probabilistic) unless empirical evidence is found to show that they must be included in the system state space.

There is a very clear hierarchy used in the state definition procedure. The activity state is the highest level followed by the commander state, followed by the angle operator state which is followed by the hardware configuration. This breakdown greatly reduces the dimensionality of the problem and reduces the number of parameters which must be identified from data.

Firing Policy

When the system is in the firing mode, the actual firing activities are accomplished according to an established policy or doctrine. The required activity states are illustrated in Figure 12. The diagram is drawn in terms of states listed in Table 3.

Description of Inputs

Actions taken by the human operators in the system presumably are based on information available to them about target actions and system performance. The required information sets have not been completely defined and cannot be until some preliminary simulation data is available. A preliminary definition for the angle operator is given below.

The basic input information is assumed to consist of the following variables listed in Table 4. Except for channel status which is a binary indicator of the operating status of the radar and optical displays, these variables are continuous. They are discretized with the following rules defined in Table 5. The procedure is straightforward with the possible exception of the angle error procedure X_A.

Basically, a four dimensional ellipsoid is used to separate the angle related inputs into in bounds and out of bounds categories.

The assignment of input symbols is accomplished with the tree structure shown in Figure 13.

Models Used in Analysis

The basic structure employed in the analysis is that of a stochastic automata with 1^{th} order states. There is one such system for each of the eight activity--commander--angle operator subsystems mentioned earlier, although in the current experiments only the range and angle operator system are considered. Each automata is represented by a set of matrices, one matrix per discrete input and time in state distribution. These matrices define the state transition probabilities for the corresponding system.

Design of Experiments

Data were collected from the AMRL simulation to meet the needs of this research. The technical sections of the protocol used for these experiments are reproduced in Appendix 4.

VII. Data Analysis Procedures

Data from the AMRL experiments are in the form of time series describing input-output sequences. It basically corresponds to the relation DM described in section II. From this data was determined the state transition matrices of describing stochastic automata. The procedures are outlined below and in the accompanying functional flow charts. The results are discussed in the next section.

The preliminary statistical analysis of the data was divided into three phases for the purposes of writing computer programs: a preprocessor transition

count routines and calculation of the parameters and statistical testing (see Figures 4, 5, and 6). A brief description of each of these phases follows.

The preprocessor uses the raw data to perform three functions: 1) determine major state (mode of operation), 2) determine the appropriate symbols for input and output signals, and 3) record the time the system is using each input/output combination. The major states used are search, acquire and track. Levels of tracking and range errors were used to determine which of these states the system is in, and parameters were collected on each major state individually.

The magnitude of the range and angle errors, as well as the operational status of the optical and radar displays, was used to define eight input symbols, each of which is defined by a given combination of these indicators. The output symbols were defined in the same manner as angle operator states in the state trees, each output symbol depending on the present major state (mode) of the system. Lastly, the length of time the present major state, input and output combination is occupied, was recorded. This information, as well as the leader record for each run, was stored for later use by the transition counting routines.

The function of the transition counting routine is to keep track of the total amount of time an input-output combination is occupied, the sum-of-squares of these times, and the number of transitions from these input-output combinations to any given output symbol. In addition, provisions were made to allow up to five old input-output combinations to be defined as the current state for purposes of counting the transitions to the next output, thus allowing us to study a fifth-order

chain of states. This information was combined for all runs of each team and trajectory combination. It was then used to calculate the system parameters.

The parameter estimation routines calculate the maximum likelihood estimate of the transitions defined under the counting portion. In addition, the variances of these values and the mean time in state were calculated, and a Chi-square test was performed to determine which chain-length has the most significance to the system under study. The option is available, at this time, to combine the transition counts of various teams or trajectories and calculate the parameters of these aggregate systems. The output of this portion consists of the estimates of the system parameters and an indication of the chain length having the most significance for this system, as well as the identifiers of the team(s) and trajectory(s) used in this estimation.

VIII. Summary of the Results

Deviations from the Protocol

Certain adjustments, dictated by hardware and resource constraints, were made in the protocol during the collection of data. The major changes included the following:

1) Use of only 7 trajectories, trajectories 1, 2, 4, 7, 8, 9, 10 as detailed in the protocol.
2) Use of four teams instead of six.
3) A total of 560 runs instead of 1200.
4) No perturbation of the range signal without simultaneous disruption of the angle display.

The first modification simply eliminated three trajectories which had characteristics very similar to other trajectories which were used and caused no major impact on the experiment or the analysis. The reduction in the number of teams was dictated by the available resources and since comparing performance across teams was not a major objective, this change also did not in itself influence the experiment. Because of time and resource constraints, the number of replications by team/trajectory combination was not increased when the above mentioned adjustments were made. This cut the total number of runs available by more than 50 percent. This did some damage to the statistical reliability of certain parameter estimates, but the objectives were to demonstrate the parameter estimation procedures rather than provide substantive data. The fourth adjustment would have been critical if substantive interpretations needed to be provided for the parameter estimates, but again, the pilot nature of this set of experiments makes this change noncritical.

Summary of the Definitions of Inputs and States

The analysis procedures followed the outline presented in sections VI and VII of this report. Inputs were defined according to the logic displayed in Figure 13. Decision criteria for the inputs are summarized in Table 6 of Appendix 7.

Major states were assigned according to Figure 3 and angle operator states were assigned according to Figures 4 and 5 with two exceptions. First, the search states were simplified to search-radar and search-optical by deleting the scan modes. Similarly, the acquire modes were simplified to acquire-radar and acquire-optical. There was also no captain state in this simulation. These adjustments were made

to simplify the data analysis and cut down data storage requirements. They have little substantive impact since behaviors were close to deterministic for all search and acquire activities. The operational definitions of all states used in the analysis are given in Tables 7 and 8 of Appendix 7.

Summary of Events Data

A summary of all events reflected in the data is given in Tables 9 and 10 of Appendix 7. Inputs two and three, particularly two, occurred substantially fewer times than the others. Input two was both optics and radar blanked and input three was radar blanked with az/el errors out of bounds. Input two seldom occurred because of the simulator configuration and subject training. If the angle operator was viewing the optical display at the time of blanking, both the optical and radar displays were blanked leaving no active display. The subjects quickly learned this fact and used primarily radar when in the auto track mode and susceptible to blanking. In this case, when the blanking was initiated only the radar system was impacted leaving the optical system available for use.

The fact that input three occurred only a small number of times reflects the fact that range errors tended to be large during blanking since no range display was available during this period. Hence, inputs four or five were the dominant ones. Also, one would expect input three to apply for a few seconds after the on set of blanking. That is, during auto track one would expect that tracking errors would be small and would remain small for a short period after blanking. This suggests that input three should occur on nearly all blanked runs, and it apparently did. The total number of blanked runs was 392 out of the total 560. There were 388 occurrences of input three.

Inputs seven and eight occurred an extraordinarily large number of times. These inputs correspond to the situation in which the radar system is operating, angle errors are in bounds and range error is in bounds or out of bounds, respectively. Because of calibration problems or sequencing of the data there was a tendency for range error to go in and out of bounds generating a large number of input change events. The exact cause is not clear, but it most likely is a difference in the limits used in range error calculations during analysis and the actual range error bounds for auto track. The data were analyzed after combining inputs seven and eight to get a better measure of decision activity.

The only state not used was major state three, state six, which corresponded to tracking in mode two with optical information. This state was not occupied for basically the same reasons given above for input two. Mode one was the preferred tracking mode for this simulation and subjects used mode two only when they forgot to switch back to mode one. (Mode switches were randomly set at the start of each run.) Furthermore, manual tracking prior to entering auto track for this first time was accomplished with radar because of the blanking characteristics discussed under input two above. If a subject forgot to reset the mode switch to mode one and attempted to enter auto track while assuming the system was in mode one, auto track would not be maintained because the angle errors would rapidly increase.

Major state one, state one, was occupied only four times showing a clear preference for the radar display during search. Acquisition was accomplished using both radar and optics, with radar the preferred mode. The other states with a low number of occurrences were major state three, states five, seven, and eight. These

all are mode two related states and were seldom occupied for the reasons given above for state six.

State transition probability estimates depend not on the number of times a state was occupied, but on the number of times a state change originated from that state, i. e., on the number of transitions. The number of transitions is substantially less than the number of occupancies because of the hierarchical structure of the model and the input dependence. That is, a major state change is not a transition and does not provide a transition count in a lower level activity state. Furthermore, an input change can result in an occupancy for two distinct configurations (major state-input-activity state) with one or perhaps no state transitions at the activity state level.

A summary of state occupations and transitions by major state-input-activity state is provided in Table 10. It is clear from this table that almost no decision activity took place when the system was in major state one, i. e., during search. Decision activity during the acquisition of a target was concentrated at inputs three and five. These are cases in which the radar was blanked and the tracking errors were out of bounds. The transitions were predominantly from state two (radar in use) to state one (optics in use).

The above events were expected. The system transitions from major state three into major state two whenever tracking errors go out of bounds. And given that the subjects generally monitor the radar scope prior to blanking, the above mentioned data simply indicate that the tracking errors would occasionally go out of bounds before the angle operator could switch from the radar display to the optics after the onset of blanking.

There was little decision activity in acquisition states other than that mentioned above.

Decision activity while tracking (major state three) centered around inputs four, seven, and to some degree eight. Input four applied after the onset of blanking, but before the errors went out of bounds. The data in Table 10 indicate that the subjects were able to make certain decisions in response to the blanking events before the tracking state was lost. Inputs seven and eight are the normal operating states with range in or out of bounds. Some small amount decision activity was exhibited by the angle operators when the angle errors were in bounds, but range was out of bounds.

Description of Transition Probability Matrices and Time in State Statistics

Table 11 contains selected transition probability matrices for the angle operator activity states for major states two and three (acquisition and track). A summary of the number of times each state was entered and the number of transitions from each state is also provided. Matrices for acquisition--major state two--contain only zeroes and over because only two states were used (radar and optics) and hence all transitions must be deterministic. Matrices are included only for those inputs which exhibited some decision activities.

Matrices for major state three exhibit some interesting behaviors. The transition from state five into state one shown in P(3, 1) reflects the change from mode two to mode one before the target appears. The fact that there were so few transitions shows that the search state was typically entered before the mode switch was made.

P(3, 2) illustrates activities for those few cases in which both displays were blanked during tracking. Subjects transitioned from manual control to auto, made display switches, and also transitioned from auto to manual. There is not sufficient data to draw many conclusions.

P(3, 4) is more instructive. This matrix applies when the radar is blanked and the tracking errors are in bounds. First of all, the presence of zeroes for states five, six, seven, and eight is an indication that mode two was not used. The first row indicates that most transitions from state one (manual control, radar display) were to state two (manual control, optics) with some to auto track. Most transitions from state two were back to state one, with some to auto track. Transitions from state four were somewhat different in the sense that most transitions were from auto to manual and not a change in display.

From the time in state summary shown in Table 12, a few additional insights are obtained. The time in state averages range from about one second to about one and one-half seconds for all states except state three, i. e., for 3, 4, 1; 3, 4, 2; 3, 4, 4 times ≈ 1.5 seconds. State 3, 4, 3 which would be the expected state just after blanking is occupied for only .7 seconds after blanking. The operator goes to a manual control state or a new display quite quickly after onset of the blanking. These times agree quite closely with those for the acquire state.

Matrix P(3, 6) is interesting in the sense that it shows what happens when the mode two mistake is made and errors drift out of bounds. Here, the auto track button has been set, but because of the operator error the system is not tracking. Recall that in mode two, the angle operator must continue to track after the auto track button is set. Transitions from state seven are generally to state three

(switch from mode two to mode one) but on 15 percent of the few transitions which occurred there was a switch in display. The time in state was also quite long, about 1.5 seconds, indicating some confusion when this state occurred.

Matrix (P3, 7) is the "normal activity" case. As above, there is some mode switch behavior apparent here. Note that in most cases in which state five was occupied (mode two, manual, radar) the transition was to state seven, (mode two, auto, radar) and in only four percent of the cases did the operators realize the mode setting error and transition to state one. Once state seven was occupied, the transitions were almost always to change mode. Transitions from states one and two were almost always to states three and nine, respectively, indicating a change from manual to automatic control. Transitions from states three and four were to states four and three, respectively, indicating switches in display but not in control. Once auto track was achieved, the system stayed there.

The time in state information for major state three, input seven, states one, two, three, and four is quite informative. (3, 7, 1) (3, 7, 2) were each occupied for about .5 seconds before a transition was made, but (3, 7, 3) was occupied for an average of 2.5 seconds with a variance of 23.81 seconds^2. Since (3, 7, 3) is the most common state, one should expect a higher mean time in state, but the high variance is interesting. One could reasonably conjecture that the time in state distribution is probably bimodal and there are two phenomena going on here. For example, behavior in this state might be quite different before and after blanking since the subjects knew the target would not be blanked a second time. They might be more willing to use the optics once the target had been reacquired after blanking and hence would have reduced times in the mode one radar tracking state.

The large variances in state (3, 7, 4) are interesting from the same perspective. More definitive answers as to why these occurred must await more complete analysis of the distribution of times in state. Such an analysis is being performed at the time of this writing.

P(3, 8) is similar to P(3, 7) in many respects. The main difference is that input eight did not occur during a manual control activity. Transitions were deterministic in the few cases observed and most transitions were changes of display. There were also a few instances in which mode two to mode one switches were made.

It was felt that because of some possible problems with the range error data that true indications of inputs seven and eight were not obtained in the above analysis. It appeared from inspection of the raw data that there were many more changes of input from seven to eight and back to seven than should have been the case. To get some feel for the sensitivity and to determine if data problems might be confounding these results, another analysis was performed. For this analysis, inputs seven and eight were collapsed essentially eliminating range error as an input. The results are presented in Table 13. There is no significant change in the transition probabilities nor in the average time in state. The variance of the time in state increased substantially, however. This seems to indicate that range error problems did not seriously impact the probability estimates and time in state estimates. Furthermore, this analysis lends some support to the hypothesis that misclassification of inputs will show up in terms of inflated variances and perhaps bimodal time in state distributions.

Tables 14 through 17 contain the transition probabilities for inputs four, seven, and eight computed for individual teams. With respect to input four there is little

difference between the teams. The biggest difference seems to be that teams three and four tended to transition from state two to state one whereas the others transitioned to state four. That means that two teams tended to switch from manual to auto control before making a display change whereas the other teams did the opposite. Either strategy is probably effective.

With input seven the teams differed only in their degree of variability. The general tendencies were the same. Teams one and three did, however, exhibit the mode two error more often than teams two and four. Under input eight the only differences between teams were the mode two errors mentioned above. Teams one and three appear to have used a much more systematized procedure and seldom were caught in mode two.

For this data at least, there appears to be no reason why aggregating data across teams should cause problems.

Tables 18 through 24 contain the state transition matrices aggregated across teams, but by individual target trajectories.

There is insufficient data to make rigorous comparisons, but there does not appear to be much difference across trajectories for input four. That is, when the display was blanked the subjects tended to do pretty much the same things.

With input seven the results are consistent across trajectories with the possible exception of state one on trajectory four. Here there was somewhat more variability with more switches from radar to optics prior to a switch from manual control to auto control than occurred on other trajectories. The sample size is not large enough to place much importance on this result, however.

Behaviors across trajectories with input eight did not vary with the exception that input eight did not occur on trajectory ten.

In general, the observed behaviors were independent of team and trajectory. Given the simplicity of the tasks, however, and the training received, this should not be unexpected. The aggregated data is therefore the best characterization of the results of the experiment.

It should also be noted that in every case tests of the hypothesis that the system was of order one versus order two resulted in the failure to reject the null. For this experiment at least, first order processes were sufficient to represent the data.

IX. Conclusions

It has been demonstrated that finite state models of human operators performing discrete control tasks can be constructed and parameters estimated from data. The methods designed in this research have been successfully applied to data obtained from a man-in-the-loop simulation. Differences in behavior under different environmental inputs have been successfully defined and parameterized using the methods developed.

A major concern with this approach was the availability of sufficient data. Although there were several input-state combinations which were not sufficiently activated during the simulation to provide appropriate data, those which were explicitly considered in the design of the experiments produced useful data even when the desired number of trials was cut by more than 50 percent. It appears that the method can be successful if the experimental tasks contain sufficient variety to

activate the states and behavior of interest. This is particularly important if the distributions of time in state is required.

Another observation relevant to the design of experiments can be made on the basis of this study. The tasks in the experiment studied here were quite predictable. For example, the subjects knew the target would be blanked only once during a given run. This had the effect of producing some artificial behavior and trapping states in the model (states which once entered were not exited). Certain other modes were simply not used (mode two). These conditions can be eliminated in future experiments.

In terms of the methodology, the most encouraging result is that the use of task dependent hierarchical decompositions of the finite state model appears to defeat the combinatorial problem of a large number of internal model states and appears also to be identifiable from data. The processing of raw tracking data from experiments is quite expensive at this point, but with better prior characterizations of the model inputs this defect can be remedied.

A lso in terms of the methodology, it appears that time in state information may also provide clues indicating possible misclassification of inputs. The definition of inputs is to some degree the most ad hoc step in the entire procedure. The development of empirical tests of the adequacy of a given set of input classifications must be developed. Sensitivity analyses can be performed, but they are enormously expensive with this type of data. More direct tests, such as the homogeneity of time in state data, are needed and, based on the results of this analysis, appear very possible.

In summary, the major theoretical and methodological hurdles have been overcome. A number of procedures must be refined and made more efficient, but we do now have the capability to identify finite state models of human operators from data.

APPENDIX 1

OUTPUT TRANSITION PROBABILITIES

Let us examine a Markov chain which has N states $\{F_1, \ldots, F_N\}$, and a transition matrix

$$P = \begin{bmatrix} P_{11} & \cdots\cdots & P_{1N} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ P_{N1} & & P_{NN} \end{bmatrix}$$

There are K different outputs $\{O_1, \ldots, O_K\}$. Output O_i is realized when the state of the underlying Markov chain is $\{F_{i_1}, \ldots F_{i_{K_i}}\}$. The initial state distribution of the Markov chain is $\Pi_0 = \{\Pi_0^1, \ldots, \Pi_0^N\}$, its distribution at time t is $\Pi_t = \Pi_0 P^t$.

<u>Definition 1.</u> $A_{ij}(O_s)$ is defined to be the transition probability of going from state i to state j, when the output resulting from state j is O_s.

<u>Theorem 1.</u> The transition matrix, when the output at time t is O_s is

$$\begin{array}{c} \begin{array}{cccc} F_{s_1} & F_{s_2} & \cdots\cdots & F_{sK_3} \end{array} \\ A(O_s) = \begin{bmatrix} 0 \ldots 0 & P_{1s_1} & 0 \ldots 0 & P_{1s_2} & \cdots\cdots & P_{1sK_s} & 0 \ldots 0 \\ \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot\ \cdot & & \cdot & \cdot\ \cdot \\ \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot\ \cdot & & \cdot & \cdot\ \cdot \\ \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot\ \cdot & & \cdot & \cdot\ \cdot \\ 0 \quad\ 0 & P_{Ns_1} & 0 \ldots 0 & P_{Ns_1} & \cdots\cdots & P_{NsK_s} & 0 \ldots 0 \end{bmatrix} \end{array}$$

<u>Proof</u>. This is clear as the new state has to have output O_s and only $\{F_{s1j} \ldots, F_{sKs}\} = F_s$ have output O_s. So P_{ij} is nonzero if and only if $j \in F_s$, and then $A_{ij} = P_{ij}$, where $\{P_{ij}\}$ is the transition matrix of the underlying Markov chain.

<u>Theorem 2.</u> The probability of an output sequence $O(1) = q_1,\ O(2) = q_3, \ldots, O(t_0) = q_{t_o}$ is given by:

$$P(O(1) = q_1, O(2) = q_2, \ldots, O(t_0) = q_{t_0} (\Pi_0) = \Pi_0 A(q_1) \cdot A(q_2) \ldots A(q_{t_0})\ e,$$

where $e^t = (1, 1, \ldots, 1)$.

<u>Proof.</u>

$$\begin{aligned} &P(O(1) = q_1, O(2) = q_2, \ldots, O(t_0) = q_{t_0} | \Pi_0) \\ &= P(O(1) = q_1 \mid \Pi_0)\ P(O(2) = q_2 \mid O(1) = q_{1j}\ \Pi_0) \\ &\qquad \ldots P(O(t_0) = q_{t_0} \mid O(1) = q_1, \ldots, O(t_0 - 1) = q_{t_0 - 1}, \Pi_0) \\ &= P(O(1) = q_1 (\Pi_0)\ P(O(2) = q_2 | O(1) = q_1) \ldots\ P(O(t_0) = q_{t_0} | O(t_0 - 1) = q_{t_0} - 1 \\ &= \Pi_0 \underbrace{A(q_1)\ A(q_2) \ldots\ A(q_{t_0})}_{A}\ e \end{aligned}$$

Π_0 gives the probability starting in state i at t = 0 and having the above outputs, $\forall$ i. Π_0 A e then gives the total probability of having the above outputs, starting in any state.

It is possible that one may not be able to observe the underlying Markov chain, but only the outputs. One might then assume that a system, whose state is $(O(t-1),\ O(t))$ is Markovian.

<u>Definition 2.</u> $q_{ij}(t) = P((O(t), O(t-1) = (j_1, j_2) \mid (O(t-1), O(t-2)) = (i_1, i_2)$, where $(i_1, i_2) = i,\ (j_1, j_2) = j$. Note $q_{ij}(t) = 0$ if $i_2 \neq j_1$.

The transition matrix for the second order output system is of the form:

$$Q(t) = [q_{ij}(t)] = \begin{array}{c|ccc:ccc:c:ccc|}
 & O_1O_1 & \cdots & O_1O_N & O_2O_1 & \cdots & O_2O_N & \cdots & O_NO_1 & \cdots & O_NO_N \\ \hline
O_1O_1 & x & \cdots & x & & & & & & & \\
O_1O_2 & & & & x & \cdots & x & & & & \\
\vdots & & & & & & & \ddots & & & \\
O_1O_N & & & & & & & & x & \cdots & x \\ \hdashline
O_2O_1 & x & \cdots & x & & & & & & & \\
O_2O_2 & & & & x & \cdots & x & & & & \\
\vdots & & & & & & & \ddots & & & \\
O_2O_N & & & & & & & & x & \cdots & x \\ \hdashline
\vdots & & & & & & & & & & \\ \hdashline
O_NO_1 & x & \cdots & x & & & & & & & \\
O_NO_2 & & & & x & \cdots & x & & & & \\
\vdots & & & & & & & \ddots & & & \\
O_NO_1 & & & & & & & & x & \cdots & x \\ \hline
\end{array}$$

__Theorem 3.__ The Q matrix is time dependent.

__Proof.__ Assume $F_{i_1} = \{F_{i_{1_1}}, F_{i_{1_2}}, \ldots, F_{i_{1_{K_{i_1}}}}\}$ give output O_{i_1}.

$F_{i_2} = \{F_{i_{2_1}}, F_{i_{2_2}}, \ldots, F_{i_{2K_{i_2}}}\}$ give output $O_{i_2} = O_{j_1}$,

and $F_{j_2} = \{F_{j_{2_1}}, F_{j_{2_2}}, \ldots, F_{j_{2_{K_{j_2}}}}\}$ give O_{j_2}.

Then

$$q_{ij}(t) = P(O(t) = j_2 \mid O(t-1) = i_1,\ O(t-2) = i_2)$$

$$= P(F(t) \in F_{j_2} \mid F(t-1) \in F_{i_1},\ F(t-2) \in F_{i_2})$$

$$= \sum_{i=1}^{k_{i_2}} \sum_{j=1}^{k_{i_1}} \sum_{k=1}^{k_{j_2}} P(F(t) = F_{j_{2_k}} \mid F(t-1) = F_{i_{1_j}},\ F(t-2) = F_{i_{2_i}})$$

$$\cdot\ P(F(t-1) = F_{i_{1_j}},\ F(t-2) = F_{i_{2_i}} \mid F(t-1) \in F_{i_1},\ F(t-2) \in F_{i_2})$$

$$= \sum_i \sum_j \sum_k P_{i_{1_j}, j_{2_k}} \frac{P(F(t-1) = F_{i_{1_j}},\ F(t-2) = F_{i_{2_i}})}{P(F(t-1) \in F_{i_{1_j}},\ F(t-2) \in F_{i_2})}$$

$$= \sum_i \sum_j \sum_k P_{i_{1_j}, j_{2_k}} \frac{P(F(t-1) = F_{i_{1_j}} \mid F(t-2) = F_{i_{2_i}})\, P(F(t-2) = F_{i_{2_i}})}{\sum_s \sum_r P(F(t-1) = F_{i_{1_s}} \mid F(t-2) = F_{i_{2_r}}) P(F(t-2) = F_{i_{2_r}})}$$

$$= \sum_i \sum_j \sum_k P_{i_{1_j}, j_{2_k}} \frac{P(F(t-2) = F_{i_{2_i}})\, P_{i_{2_i}, i_{1_j}}}{\sum_s \sum_r P(F(t-2) = F_{i_{2_r}})\, P_{i_{2_r}, i_{1_s}}}$$

So the transition matrix for a second order output system is time dependent, for all values of t.

Corollary. The transition matrix for an n-th order output system is time dependent for all values of t.

Proof. This can be proved in exactly the same manner as theorem 3.

Theorem 4. For a second order output system.

$$q_{ij}(t) \le \sum_{i \in F_{i_1}} \sum_{j \in F_{i_2}} \sum_{k \in F_{j_2}} (P_{ij}\, P_{jk}) \left(1 + \frac{A}{A+B(t-2)} \left(\frac{E_i(t-2)}{\Pi_i} - \frac{B(t-2)}{A} \right) \right)$$

where $P(F(t) = i) = \Pi_i t\, E_i(t) \xrightarrow[t \to \infty]{} \Pi_i$

$$A = \sum_{n \in F_{i_1}} \sum_{m \in F_{i_2}} \Pi_n P_{nm}, \quad B(t) = \sum_{n \in F_{i_1}} \sum_{m \in F_{i_2}} E_n(t)\, P_{nm}.$$

Proof.

$$q_{ij}(t) = \sum_i \sum_j \sum_k P_{ij}\, P_{jk} \frac{P(F(t-2) = i)}{\sum_n \sum_m P(F(t-2) = n) P_{nm}}$$

$$= \sum_i \sum_j \sum_k P_{ij}\, P_{jk} \frac{\Pi_i + E_i(t-2)}{A + B(t-2)}$$

$$\le \sum_i \sum_j \sum_k P_{ij}\, P_{jk} \frac{\Pi_i + E_i(t-2)}{A + B(t-2)} \quad \frac{A}{\Pi_i}$$

$$= \sum_i \sum_j \sum_k P_{ij}\, P_{jk} \frac{A\Pi_i + B\Pi_i + E_i A - \Pi_i B(t-2)}{\Pi_i (A + B(t-2))}$$

$$= \sum_i \sum_j \sum_k P_{ij}\, P_{jk} \left(1 + \frac{A}{A + B(t-2)} \left(\frac{E_i(t-2)}{\Pi_i} - \frac{B(t-2)}{A} \right) \right)$$

Lemma. If $|E_i(t_0-1)| = A$, then $|E_i(t_0)| \leq A\lambda$ max

where λ_{max} is the maximum eigenvalue of the matrix P.

Proof. $E_i(t_0-1) = \sum_n a_n \lambda^{t_0-1} = A$

$$E_i(t_0) = \sum_n a_n A_n^{t_0} = \sum_n a_n \lambda_n^{t_0-1} \lambda_n$$

$$\leq \sum_n \lambda_n^{t_0-1} \lambda_{max} = A \lambda_{max}$$

This is true because all the eigenvalues are positive between 0 and 1.

Definition 3. $\eta \quad \eta(t) = \max_i \left| \frac{E_i(t)}{\Pi_i} \right|$

Lemma 2. $\left| \frac{B(t)}{A} \right| \leq \eta(t)$

Proof. Trivial.

Definition 4. $K_0(t) = \min_{i \in F_{i_1}} \frac{E_i(t)}{\Pi_i}$, $K_1(t) = \max_{i \in F_{i_1}} \frac{E_i(t)}{\Pi_i}$

Note $\eta(t) = \max(|K_0(t)|, |K_1(t)|)$

Lemma 3.

$$\frac{A}{A + B(t)} \leq \frac{1}{1 + K_0(t)}$$

Proof.

$$\frac{A}{A + B(t)} \leq \frac{1}{1 + B(t)/A} \leq \frac{1}{1 + K_0(t)}$$

Theorem 5. $q_{ij}(t) \leq \sum_i (\sum_j \sum_k P_{jk} P_{ij}) \left(1 + \frac{2\eta(t-2)}{1 + K_0(t-2)} \right)$

Proof. Clear from lemmas 3 and 4.

Corollary 2. For an n-th order output system one has

$$q_{i_1,\ldots,i_n}(t) \leq \sum_{i_1}\left(\sum_{i_2}\cdots\sum_{i_n} P_{i_1 i_2} P_{i_2 i_3}\cdots P_{i_{n-1}, i_n}\right)\left(1+\frac{A}{A+B(t-2)}\left(\frac{E_i(t-2)}{\Pi_i}-\frac{B(t-2)}{A}\right)\right)$$

where $A = \sum_{i_1}\cdots\sum_{i_{n-1}} \Pi_{i_1} P_{i_1 i_2}\cdots P_{i_{n-2}, i_{n-1}}$, as

$$B(t) = \sum_{i_1}\cdots\sum_{i_{n-1}} E_{i_1}(t)\, P_{i_1 i_2}\cdots P_{i_{n-2}, i_{n-1}}$$

and $P(F(t) = i) = \Pi_i + E_i(t) \xrightarrow[t\to\infty]{} \Pi_i$

<u>Proof.</u> This is obtained in a similar manner as theorem 5.

<u>Theorem 6.</u>

1) $B(t) > 0,\ \forall t$ then $\dfrac{2\eta(t-2)}{1+K_0(t-2)} < 2\lambda_{max}^{t-2}$

2) $B(t) < 0,\ \forall t$ then $\dfrac{2\eta(t-2}{1+K_0(t-2)} < \lambda_{max}^{t-2}$

3) $B(t) > 0 \quad t \leq t^*$ then $\dfrac{2\eta(t-2)}{1+K_0(t-2)} \leq 2\lambda_{max}^{t-2}$
   $B(t) > 0 \quad t > t^*$

4) $B(t) < 0 \quad t \leq t^*$ then $\dfrac{2\eta(t-2)}{1+K_0(t-2)} \leq \lambda_{max}^{t-2}$
   $B(t) > 0 \quad t > t^*$

<u>Proof.</u>

1) $$\frac{2\eta(t-2)}{1+K_0(t-2)} < \frac{2\eta(0)\lambda_{max}^{t-2}}{1+K_0(t-2)} < \frac{2\eta(0)\lambda_{max}^{t-2}}{1} \leq 2\lambda_{max}^{t-2}$$

as $K_0(t-2) > 0,\ \forall t$ and

$$\frac{2\eta(0)}{1+K_0(0)} \le 1 \Rightarrow 2\eta(0) \le (1+K_0(0)) \le 2.$$

2) $$\frac{2\eta(t-2)}{1+K_0(t-2)} \le \frac{2\eta(0)\lambda_{max}^{t-2}}{1+K_0(0)} \le \lambda_{max}^{t-2}$$

as $$\frac{2\eta(0)}{1+K_0(0)} \le 1$$

3) $$\frac{2\eta(t-2)}{1+K_0(0)} \le 2\eta(0)\lambda_{max}^{t-2} \le 2\lambda_{max}^{t-2}$$

as $$\frac{2\eta(t-2)}{1+K_0(t-2)} \le \frac{2\eta(t-2)}{1-|K_0(t-2)|} \le \frac{2\eta(0)\lambda_{max}^{t-2}}{1-|K_0(0)|} \le \lambda_{max}^{t-2}$$

4) $$\frac{2\eta(t-2)}{1+K_0(t-2)} \le \frac{2\eta(t-2)}{1+K_0(0)} \le \frac{2\eta(0)\lambda_{max}^{t-2}}{1+K_0(0)} \le \lambda_{max}^{t-2}$$

Theorem 7. The proportionate error at time t when one uses Π_i instead of $\Pi_i + E_i(t)$ is no greater than $2\lambda_{max}^{t-2}$.

Proof. See theorem 6.

APPENDIX 2

A COMPUTATIONAL EXAMPLE

The system to be examined can be modeled by a first order Markov chain with transition matrix Q. The state space is $\{1, 2, 3, 4, 5, 6\}$.

$$Q = \begin{bmatrix} 0.10 & 0.20 & 0.25 & 0.15 & 0.05 & 0.25 \\ 0.60 & 0.10 & 0.05 & 0.05 & 0.10 & 0.10 \\ 0.20 & 0.10 & 0.15 & 0.05 & 0.40 & 0.10 \\ 0.10 & 0.10 & 0.10 & 0.10 & 0.15 & 0.15 \\ 0.50 & 0.05 & 0.05 & 0.20 & 0.10 & 0.10 \\ 0.05 & 0.15 & 0.30 & 0.40 & 0.05 & 0.05 \end{bmatrix}$$

States 1 and 2 are assumed to give output A, states 3 and 4 give output B, and states 5 and 6 give output C. Q was used to simulate the system transitions for 1000 time units. Assuming the output transitions can be modeled by a first order Markov chain, the transition count for that chain is N_1.

$$N_1 = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 223 & 114 & 112 \\ 129 & 61 & 108 \\ 96 & 124 & 34 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \end{array}$$

Maximum likelihood estimates of the transition probabilities were developed. The transition matrix is P_1

$$P_1 = \begin{bmatrix} 0.49666 & 0.25390 & 0.24944 \\ 0.43289 & 0.20470 & 0.36242 \\ 0.37795 & 0.48819 & 0.13386 \end{bmatrix}$$

From N_1, $\hat{Q}_1$ a maximum likelihood estimate of the transition matrix Q of the state system is developed.

$$\hat{Q}_1 = \begin{bmatrix} 0.2485 & 0.2485 & 0.1270 & 0.1270 & 0.1245 & 0.1245 \\ 0.2485 & 0.2485 & 0.1270 & 0.1270 & 0.1245 & 0.1245 \\ 0.2165 & 0.2165 & 0.1025 & 0.1025 & 0.1810 & 0.1810 \\ 0.2165 & 0.2165 & 0.1025 & 0.1025 & 0.1810 & 0.1810 \\ 0.1890 & 0.1890 & 0.2440 & 0.2440 & 0.0670 & 0.0670 \\ 0.1890 & 0.1890 & 0.2440 & 0.2440 & 0.0670 & 0.0670 \end{bmatrix}$$

$\hat{Q}$ is then used to simulate the system and the output transitions are modeled with a Markov chain. The transition count for the outputs is

$$\hat{N}_1 = \begin{bmatrix} 242 & 108 & 108 \\ 132 & 75 & 99 \\ 84 & 123 & 29 \end{bmatrix}$$

From this $\hat{P}_1$ a maximum likelihood estimate of the output transition matrix is developed.

$$\hat{P}_1 = \begin{bmatrix} 0.52941 & 0.23529 & 0.23529 \\ 0.43137 & 0.24510 & 0.32353 \\ 0.35593 & 0.52119 & 0.12288 \end{bmatrix}$$

The null hypothesis that N_1 comes from $\hat{P}_1$ can be tested against the alternative hypothesis that it comes from P_1

$$\lambda = \max \frac{L_{\hat{P}_1}}{L_{P_1}} = 0.038$$

Is is known that

$$-2 \ln \lambda \simeq \chi_3^2$$

$$-2 \ln \lambda = 6.5$$

$$\chi_3^2 (0.05) = 7.81$$

So we accept the null hypothesis. Therefore one can not say that N_1 could not have been generated by $\hat{P}_1$.

Now let us assume that the output transitions cannot be modeled with a first order Markov chain, but it can be modeled by a second order Markov chain. The transition for the second order output system is N_2.

$$N_2 = \begin{bmatrix} 112 & 58 & 53 \\ 47 & 16 & 51 \\ 40 & 62 & 10 \\ 75 & 26 & 28 \\ 23 & 19 & 18 \\ 46 & 42 & 20 \\ 35 & 30 & 31 \\ 59 & 26 & 39 \\ 10 & 20 & 9 \end{bmatrix}$$

From N_2, maximum likelihood estimates of the second order output transition probabilities are developed. The transition matrix is P_2

$$P_2 = \begin{bmatrix} 0.52335 & 0.26009 & 0.23767 \\ 0.41228 & 0.14035 & 0.44737 \\ 0.35714 & 0.55357 & 0.08929 \\ 0.58140 & 0.20155 & 0.21705 \\ 0.38333 & 0.31667 & 0.30000 \\ 0.42593 & 0.38889 & 0.18519 \\ 0.36458 & 0.31250 & 0.32292 \\ 0.47581 & 0.20968 & 0.31452 \\ 0.29412 & 0.58824 & 0.11765 \end{bmatrix}$$

From N_2, $\hat{Q}_2$ a maximum likelihood estimate of the second order transition matrix of the underlying system can be developed. $\hat{Q}_2$ is then used to simulate the system. The output transitions are modeled with a second order Markov chain. The transition count for the output is

$$\hat{N}_2 = \begin{bmatrix} 143 & 59 & 54 \\ 46 & 19 & 41 \\ 41 & 61 & 10 \\ 76 & 26 & 25 \\ 22 & 18 & 21 \\ 41 & 39 & 18 \\ 36 & 21 & 33 \\ 60 & 24 & 36 \\ 8 & 20 & 2 \end{bmatrix}$$

From this $\hat{P}_2$ a maximum likelihood estimate of the output transition matrix is developed.

$$\hat{P}_2 = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.55859 & 0.23047 & 0.21094 \\ 0.43396 & 0.17925 & 0.38629 \\ 0.36609 & 0.54464 & 0.08929 \\ 0.59843 & 0.20472 & 0.19685 \\ 0.36066 & 0.29508 & 0.34426 \\ 0.41837 & 0.39796 & 0.18367 \\ 0.40000 & 0.23333 & 0.36667 \\ 0.50000 & 0.20000 & 0.30000 \\ 0.26667 & 0.66667 & 0.06667 \end{bmatrix} \end{array} \begin{array}{c} \\ AA \\ AB \\ AC \\ BA \\ BB \\ BC \\ CA \\ CB \\ CC \end{array}$$

The null hypothesis that N_2 comes from P_2 can be tested against the alternative hypothesis that it comes from P_2

$$\lambda = \max \frac{L_{\hat{P}_2}}{L_{P_1}} = 0.3546$$

It is known that

$$-2 \ln\lambda \simeq \chi_9^2$$

$$-2 \ln\lambda \simeq 2.07 < \chi_9^2\ (0.05) = 11.4$$

So we accept the null hypothesis. Therefore, one can not say that N_1 could not have been generated by $\hat{P}_2$.

The first order output transition matrix generated by $\hat{Q}_2$ is $\hat{\hat{P}}_1$

$$\hat{\hat{P}}_1 = \begin{bmatrix} 0.54008 & 0.22363 & 0.23629 \\ 0.44599 & 0.21254 & 0.34146 \\ 0.37500 & 0.50000 & 0.12500 \end{bmatrix}$$

If one tests the null hypothesis that N_1 came from $\hat{\hat{P}}_1$ against the alternative hypothesis that it came from P_1.

$$\lambda = \frac{\max L_{\hat{\hat{P}}_1}}{\max L_{P_1}} = 0.104$$

$$-2 \ln \lambda \simeq 2.26 < \chi_3^2 \; (0.05) = 7.8)$$

So $\hat{\hat{P}}_1$ is preferred to $\hat{P}_1$.

From those results the following statement can be made:

If one can only observe the outputs of a system, but not the real states, the output transitions can be modeled as a Markov chain. A transition matrix can be developed and from this matrix a transition matrix of the underlying system can be inferred. Then one cannot reject the hypothesis that the output sequence generated by the real system was generated by the inferred transition matrix.

APPENDIX 3

ESTIMATION OF TRANSITION PROBABILITIES

The problem to be examined here is the estimation of transition probabilities for a finite state Markov chain. The state of the chain at time t is defined to be a two dimensional vector $S(t) = (I(t), F(t))$, where $I(t)$ is an input to the system at time t and $F(t)$ is the state of the system at time t.

One is interested in the probability that the state of the chain is S_t at time t given it is past, or:

$$P(S(t) = S_t \, (S(t-1) = S_{t-1}, \ldots, S(0) = S_0). \qquad (1)$$

Under the assumption that the system can be represented by a first order Markov chain, (1) reduces to

$$P(S(t) = S_t(S(t-1) = S_{t-1}), \qquad (2)$$

if $S_t = (i, m)$ and $S_{t-1} = (j, n)$, (2) can be written as

$$P_{imjn} = P(F(t) = j, \; I(t) = n \mid F(t-1) = i, \; I(t-1) = m). \qquad (3)$$

Let us assume that, $1 \leq i, j \leq N$, $1 \leq n, m \leq K$. Then there exists a mapping X which maps the two dimensional state vector (i, m) into a one dimensional state vector (g),

$$X(i, m) = m \cdot N + i = g. \qquad (4)$$

Let us assume $X(S(t)) = G(t)$, then (3) is equivalent to

$$P_{gn} = P(G(t) = h \mid G(t-1) = g),$$

when $X(i, m) = g$ and $X(j, n) = h$.

Now P_{gn} can be estimated using the known theory of estimation of transition probabilities for a Markov chain. This theory is developed in a paper by Anderson

and Goodman (1).

J realizations of the system transitions are observed. The transition count is $\{n_{gh}(t)\}_{ght}$, where $n_{gh}(t)$ is the number of transitions from g to h at time t, $(G(t-1) = g,\ G(t) = h)$. Realization j is observed for t_j+1 time units starting at time 0. The total number of observations made at time t is M(t); thus

$$\sum_{gh} n_{gh}(t) = M(t), \quad 0 \le n_{gh}(t) \le M(t) \le J.$$

The maximum likelihood estimate of P_{gh} is

$$\hat{P}_{gh} = \frac{\sum_{t=1}^{t_{max}} n_{gh}(t)}{\sum_{h=1}^{NK} \sum_{t=1}^{t_{max}} n_{gh}(t)}, \quad \forall\, g, h \tag{5}$$

Now one is mainly interested in $P_{imj} = P(F(t) = j \mid F(t-1) = i,\ I(t-1) = m)$, the probability that the state of the system changes from i at time t-1 to j at time t given an input m at time t-1.

Keeping (2) shows that if $\hat{\theta}$ is a maximum likelihood estimate of θ and f is a continuous function, then the maximum likelihood estimate of $f(\theta)$ is $f(\hat{\theta})$.

It is clear that

$$P_{imj} = \sum_{k=1}^{K} P_{(i+Nm)(j+NK)}, \quad 1 \le i,\ j \le N,\ 1 \le m \le K,$$

therefore the maximum likelihood estimate of P_{imj} is

$$\hat{P}_{imj} = \sum_{k=1}^{K} \hat{P}_{(i+Nm)(j+NK)} \quad 1 \le i,\ j \le N,\ 1 \le m \le K. \tag{6}$$

When one does not know with certainty the correct order of the Markov chain under study, one is interested in testing the null hypothesis, that the chain

is of order K, against the alternative hypothesis, that its order is K + 1.

If K = 1, the transition probabilities assuming the null hypothesis is true, are

$$P_{gh} = P(G(t) = h \mid G(t-1) = g), \forall g, h.$$

If the alternative hypothesis is true, the transition probabilities are

$$P_{fgh} = P(G(t) = h \mid G(t-1) = g, \; G(t-2) = f) \quad \forall f, g, h .$$

The maximum likelihood estimate of P_{fgh} is of the same form as $\hat{P}_{gh}$.

$$\hat{P}_{fgh} = \frac{\sum_{t-2}^{t_{max}} n_{fgh}(t)}{\sum_{h=1}^{NK} \sum_{t=2}^{t_{max}} n_{fgh}(t)}, \quad \forall f, g, h, \tag{7}$$

where $n_{fgh}(t)$ is the number of realizations which occupied state f at t-2, state g at time t-1, and state h at time t. Then it has been shown that $-2 \log \lambda$ has an asymptotic χ^2 distribution with $NK(NK-1)^2$ degrees of freedom, where

$$\lambda = \prod_{f=1}^{NK} \prod_{g=1}^{NK} \prod_{h=1}^{NK} \left(\frac{\hat{P}_{gh}}{\hat{P}_{fgh}} \right)^{n_{fgh}} \qquad (n_{fgh} = \sum_{t=2}^{t_{max}} n_{fgh}(t)) \tag{8}$$

This generalizes to K > 1, very easily.

As one is mainly interested in

$$\begin{aligned} P_{imj} = P_{gj} &= P(F(t) = j \mid F(t-1) = i, \; I(t-1) = m) \\ &= P(F(t) = j \mid G(t-1) = g) \end{aligned} \tag{9}$$

one would compare (9) to

$$P_{fgh} = P(F(t) = j \mid G(t-1) = g,\ F(t-2) = f).$$

$-2 \log \lambda$ then has an asymptotic χ^2 distribution with NK(NK-1)(N-1) degrees of freedom, where

$$\lambda = \prod_{f=1}^{NK} \prod_{g=1}^{NK} \prod_{j=1}^{NK} \left(\frac{\hat{P}_{gj}}{\hat{P}_{fgj}} \right)^{n_{fgj}} \tag{10}$$

Having obtained estimates of the transition probabilities, it is desirable to construct confidence intervals around those estimates. It is known that $\hat{P}_{gh}$ is asymptotically normally distributed with the following second order moments.

$$E(\hat{P}_{gh}) = P_{gh}$$

$$\mathrm{Var}\,(\hat{P}_{gh}) = \frac{\hat{P}_{gh}(1 - \hat{P}_{gh})}{\hat{n}_g}, \qquad g^* = \sum_{g=1}^{NK} \sum_{t=1}^{t_{max}} gh(t)$$

$$\mathrm{Cov}\,(\hat{P}_{gh}, \hat{P}_{rs}) = \begin{cases} \dfrac{-P_{gh}P_{gs}}{n_{g^*}} & \text{if } g = r, \\ 0 & \text{otherwise.} \end{cases}$$

From this one obtains the moments of $\hat{P}_{gj}$,

$$E(\hat{P}_{gj}) = P_{gj}$$

$$\mathrm{Var}\,(\hat{P}_{gj}) = \mathrm{Var}\,\Big(\sum_{m=1}^{K} P_{g(jm)} \Big) = \sum_{n=1}^{K} \mathrm{Var}\,(\hat{P}_{g(jn)})$$

$$+ \sum_{\substack{r=1 \\ r \neq s}}^{K} \sum_{s=1}^{K} \mathrm{Cov}\,(\hat{P}_{g(jr)}, \hat{P}_{g(js)})$$

The central limit theorem then gives a $(1 - \alpha) \times 100\%$ confidence interval of P_{gj} as

$$(\hat{P}_{gj} \pm Z_{\alpha/2} \cdot \sqrt{\text{var} \ (\hat{P}_{gj})}).$$

REFERENCES

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APPENDIX 4

PROTOCOL

A. Objectives

1. Determine the degree to which subjects can perform discrete control tasks on the simulator apparatus.

2. Determine the input characteristics required to induce subjects to execute discrete control options during a run.

3. Provide data for testing discrete control model parameter estimation algorithms.

4. Provide data for preliminary analysis of relationships between system mode of operation, target trajectory, control configuration and information channel configuration.

5. Evaluate alternative scoring metrics for assessing discrete control performance.

B. Background

Previous manual control studies have emphasized continuous tracking of azimuth, elevation and range. This study is concerned with an entirely different set of information processing and control activities, namely, how operators configure the system under their control and how they utilize available input information channels. It is the first step in modeling the supervisory control decisions and the corresponding coordinating information flows in a multilevel man-machine system.

C. Relevance

This experiment is basically a pilot study which serves to further clarify the modeling issues in the discrete control problem. The data provided will serve

to test parameter estimation algorithms and provide statistical information for better experimental design.

Ultimately discrete control experiments will produce data and models which can be used to enrich threat quantification models through incorporation of the resource management and system configuration decisions.

D. Experimental Plan

1. Apparatus

The dynamics of the simulator are designed to represent the following AAA functions:

a. Use of left hand optics slaved to the radar antenna.

b. Use of the PPI scope for angle tracking by radar.

c. Use of A scope for range tracking.

d. Use of sector scan, circular scan and fast circular scan radar modes for target detection and acquisition.

e. Manual azimuth, elevation and range tracking.

f. Manual azimuth, elevation tracking with automatic range tracking (mode 2).

g. Automatic azimuth, elevation and range tracking (mode 1).

2. Subjects

A total of 12 trackers will be used for this study. Subjects will be used in pairs, one tracker serving as range operator, one as the angle operator. The subjects are male and female ranging in age from 18 to 25 years. Their vision is 20/20 and they may wear corrective lenses to insure this. They receive monetary payment for time spent tracking during experiments and during training.

Although the trackers vary in their proficiency with tracking devices, all will be trained to an acceptable level of performance as determined by the training supervisor and principal investigator. Given the lack of experience

with discrete control tasks during tracking, it is not clear that a single measure can be used to assess proficiency, but minimum performance must include the following

a. Acquisition of the target on all trajectories within 30 seconds of target introduction.

b. Enter auto track mode on all trajectories.

c. Ability to use mode 1 on all trajectories.

d. Ability to switch from optical to radar input upon blanking of screen (angle operator).

e. Switch from radar to optical input upon loss of radar return.

f. Detect loss of auto track, reacquire target and reinitialize auto track.

During training the trackers are free to use either eye for optical angle tracking and may select any magnification or filter in the optical system. The angle operator may select any control configuration and any radar configuration (sector scan, fast circular scan, circular scan). The range operator may also select any control configuration. Subjects should be encouraged to exercise all discrete control options and completely familiarize themselves with the capabilities of the system. During the early phases of training the trajectories should not be varied from run to run. Teamd should practice on a given trajectory until performance is acceptable. When all trajectories have been mastered with no blanking of the displays or disruption of auto track, blanking should be introduced on the radar displays and optical displays on about one half of the practice runs in the training session. Optical blanking and radar blanking should not be performed on the same run. When subjects have attained an acceptable level performance on this task when the trajectory is known beforehand, all ten trajectories should be used in a given training session, randomly ordered and with randomly placed blanking of the displays. That is the format of the actual experiment should be used. Mode 1 and mode 2 settings should be randomly made at the beginning of each run.

Trackers will require approximately 4 weeks for training. They will be utilized for a period of 20 days for data collection. Each team will be used for approximately 45 minutes each during both training and data collection for the experiment. Each 45 minute session will involve tracking 20 trajectories, each of which is approximately 60 seconds in duration. Appropriate rest periods will be given between trajectories.

3. Design

During the experiment there will be ten different trajectories administered for the subjects to track. The trajectories are described in Table 1. These trajectories are the same as trajectories used in previous tracking studies with the exception that 20 seconds of straight and level flight have been concatenated to the beginning of each trajectory to accomodate detection and acquisition of the target.

The following variables will be recorded during the experiment:

a. Manual notes:

Trackers' comments on strategy used.

Experimenter's observations.

Training supervisor's observations.

b. Automatic data acquisition:

Tracking data:

Target trajectory including azimuth, elevation, range, roll, pitch and yaw.

Azimuth tracking error

Elevation tracking error

Range tracking error

Indication of blanking of the display on any input channel

Discrete data:

Radar configuration
circular scan
fast circular scan
sector scan

Sight configuration
2x magnification, clear filter
2x magnification, neutral density filter
2x magnification, orange filter

6x magnification, clear filter
6x magnification, neutral density filter
6x magnification, orange filter

Angle control configuration
Azimuth rate, elevation rate
Azimuth position, elevation position
Azimuth position, elevation rate
Azimuth rate, elevation position

Range control configuration
Coarse
Fine

Mode of operation
Mode 1
Mode 2

Tracking mode
Manual
Automatic
Search

Angle operator input channel
Optical (viewing through the sight)
Radar (not viewing through the sight)

These data will be recorded as time series for purposes of analyzing discrete control performance and providing feedback to the trackers. Data tapes will contain all tracking data and discrete data recorded at a rate of at least 10 Hz.

Feedback for the trackers is displayed on the auxiliary CRT after each trajectory is run. The displayed information includes:

Team number

Run number

Total time out of auto track

In planning the experimental treatments the following resources and guidelines were considered:

20 workdays maximum

6 teams of 2 trackers each

10 trajectories

3-6 teams per day

20 runs per team per day

1 tracking session per team per day

at least 20 replication per trajectory per team

The application of experimental treatments involves the following rules:

a. Each team will track each of the ten trajectories twice during a given session.

b. The order of presentation of trajectories during a session will be randomized.

c. Over the course of the experiment each trajectory will be tracked 20 times.

d. Seven runs with each trajectory will involve a perturbation of the range signal forcing the target out of the range gate with simultaneous opening of the auto track loop. The angle displays are not otherwise disturbed.

e. Seven runs with each trajectory will involve disruption of the angle displays and opening of the auto track loop. If the system is in mode 1, the radar screen will be saturated. If the system is in mode 2, the optical display will be blanked.

f. The remaining trajectories will not be perturbed.

g. Introduction of disturbances will be randomized over the 20 runs.

h. Duration of the disturbance will be random (uniformly distributed) over the interval 4 seconds to 10 seconds or the end of the run whichever comes first.

i. The point of introduction of a disturbance will be random over the interval 5 to 15 seconds after acquisition of auto track, but never within 10 seconds of the end of a run.

j. If auto track is not achieved, or is not achieved in sufficient time, on a trajectory which was to be perturbed, the trajectory will be repeated if at all possible.

A total of 1200-2400 total runs will be available from the experiment. During the course of the experiment and training, progress will be monitored to determine if tradeoffs are warranted in the number of sessions, trajectories, replications and procedures of disturbance introduction.

4. Procedures

Standard warm up procedures will be followed.

The sector into which the target will be introduced will be communicated to the angle operator. Sector widths of approximately 45 degrees will be used. The initial mode setting will also be communicated to him at this time. Mode 1 and mode 2 settings will be randomly generated. The angle operator will signal that the team is ready by depressing a foot s switch. The target will then appear in the predetermined sector after a random delay of 4 to 10 seconds. After each run feedback to the team will be displayed on the auxiliary CRT and short rest will be given. The procedure will then be repeated for the next trajectory. Whenever feasible, runs in which a disturbance was planned and in which auto track was not achieved will be repeated to obtain the planned number of replications.

At the completion of each session, the team will be given the opportunity to make comments regarding strategy or other noteworthy apsects of the session.

Trajectory		Approx. Crossover Conditions	Altitude (m)	Airspeed (m/sec)	Maximum Angular Rate		Duration (seconds)
Number	Name	a. offset (m) b. altitude (m) c. slant range (m)			Azimuth (deg/sec)	Elevation (deg/sec)	
1	Flyby 2 × 2	a. 609 b. 609 c. 861	609	183	17	4	55
2	Flyby 0 × 5	a. "152.4" b. "1524" c. "1531"	1524	213	0	17.8	35
3	Flyby 4 × 5	a. 1219 b. 1524 c. 1951	1524	213	10	2.5	55
4	Flyby 0 × 1.6	a. "381" b. "490" c. "619"	490	198	0	11.9	35
5	Flyby 4 × 1.6	a. 1219 b. 490 c. 1314	490	198	9.5	1.5	55
6	Flyby 4 × .3	a. 1219 b. 91 c. 1222	91	198	9.5	.3	55

TABLE 1 CHARACTERISTICS OF TRAJECTORIES

Trajectory		Approx. Crossover Conditions	Altitude (m)	Airspeed (m/sec)	Maximum Angular Rate		Duration (seconds)
Number	Name	a. offset (m) b. altitude (m) c. slant range (m)			Azimuth (deg/sec)	Elevation (deg/sec)	
7	FAIR PASS	a. 201 b. 678 c. 707	614-2268	152-237	59	15	55
8	WEAPON DELIVERY (14-3)	a. 671 b. 158 c. 689	122-194	155-259	23.1	2.5	55
9	RECON (14-1)	a. 1122 b. 535 c. 1243	304-638	155-202	8.6	3.1	55
10	ZIG-ZAG (14-2)	a. 1029 b. 512 c. 1149	399-920	101-164	16.2	6.7	55

NOTES:

1. Trajectories No. 2 and No. 4 do not actually achieve "crossover" due to hardware limitations. Values shown in quotes represent the conditions closest to the crossover, the end of the trajectory.
2. Figures are approximates.
3. Flyby "X × Y" refers to offset and altitude, respectively, in thousands of feet.
4. "Crossover" is defined as the point of minimum offset.

TABLE 1. CHARACTERISTICS OF TRAJECTORIES

APPENDIX 5

TABLES REFERRED TO IN TEXT

Table 1

Indicator #	Value	Enabling Condition
1	1	Optical data and left site configuration
	0	Otherwise
2	1	System in mode 4 operation
	0	Otherwise
3	1	Radar data to angle operator
	0	Otherwise
4	1	All nontracking activity states
	0	Tracking states
5	1	Az control in position configuration
	0	Otherwise
6	1	El control in position configuration
	0	El control in rate configuration
7	1	Manual track or (auto track and mode 2) of mode 4
	0	Otherwise
8	1	Mode 1 and auto track
	0	Otherwise
9	1	Radar in circular scan mode
	0	Otherwise
10	1	Radar in fast circular scan mode
	0	Otherwise
11	1	Radar in sector search mode
	0	Otherwise
12	1	Range control set to coarse
	0	Range control set to fine
13	1	Auto track
	0	Otherwise
14		Not used

Table 1 (Continued)

Indicator #	Value	Enabling Condition
15	0	Reflex site slaved to antenna--system overridden by the commander
	1	Otherwise
16	1	Fire control with angle operator
	0	Fire control with commander
17	1	Gunner's coolant switch depressed
	0	Otherwise
18	1	Gunner's trigger depressed
	0	Otherwise
19	1	Commander's coolant switch depressed
	0	Otherwise
20	1	Commander's trigger depressed
	0	Otherwise
21	1	Interlock system shunted
	0	Otherwise
22	1	Upper guns activated
	0	Upper guns not activated
23	1	Lower guns activated
	0	Lower guns not activated
24	1	Commander viewing through reflex site
	0	Otherwise
25	1	Commander viewing scene directly
	0	Otherwise

Table 2

Unit of Description	States	Controlled by*
Major system function	search acquire track fire	1, 3
System mode	1 2 3 4	3
Computer input channel	reflex site antenna and site	3
Commander info channel	reflex site direct observation computer output displays	3
Angle information channel	optical radar	1
Tracking mode	manual auto	1
Firing configuration	angle operator commander	3
Gun configuration	upper only lower only both	3
Interlock	shunted not shunted	3
Radar	circular scan fast circular scan sector scan	1
Az tracking control	rate position	1
El tracking control	rate position	1
Range tracking control	coarse fine	2
Left sight	see Figure 10	1
Right site	see Figure 11	1

* 1 denotes angle operator
2 denotes range operator
3 denotes commander

Table 3

Firing Policy States:

Local Cumulative States:

State 1 ---Total less than max allowed*

State 2---Total greater than max

Burst States:

State 1---short	3-10	rds
State 2---long	10-20	rds
State 3---cont	20-50	rds

Cooling States:

State 1---short	0.5-1	sec
State 2---inter	2-3	sec
State 3---major	10-15	sec

---

*The maximum is 120-150 rds.

Firing States: Defined from cumulative states and burst states

State (1, 1)	Total less than max and short burst
State (1, 2)	Total less than max and long burst
State (1, 3)	Total less than max and cont burst
State (2, 1)	Total greater than max and short burst
State (2, 3)	Total greater than max and cont burst

Table 4

Input Variable Definitions

Variable	Symbol
Range	R
Azimuth Error	AE
Elevation Error	EE
Azimuth Error Rate	AER
Elevation Error Rate	EER
Range Error	RE
Channel Status	CS

Table 5

Random Variable Definitions

Definition	Properties
X_R: R $\rightarrow$ $\{0, 1\}$	$X_R(r) = \begin{cases} 1 & r > Rmax \\ 0 & r \leq Rmax \end{cases}$
X_A: AE x EE x AER x EER $\rightarrow$ $\{0, 1\}$	$X_A(\alpha, \beta, \gamma, \delta) = \begin{cases} 0 & a_1\alpha^2 + a_2\beta^2 + a_3\gamma^2 + a_4\delta^2 < 1 \\ 1 & \text{otherwise} \end{cases}$
X_{RE}: RE $\rightarrow$ $\{0, 1\}$	$X_{RE}(\alpha) = \begin{cases} 0 & \alpha < rmax \\ 1 & \text{otherwise} \end{cases}$
X_{CS}: CS $\rightarrow$ $\{0, 1, 2\}$	$X_{CS}(\alpha) = \begin{cases} 0 & \text{both channels OK} \\ 1 & \text{radar down} \\ 2 & \text{optics down} \end{cases}$

APPENDIX 6

FIGURES REFERRED TO IN TEXT

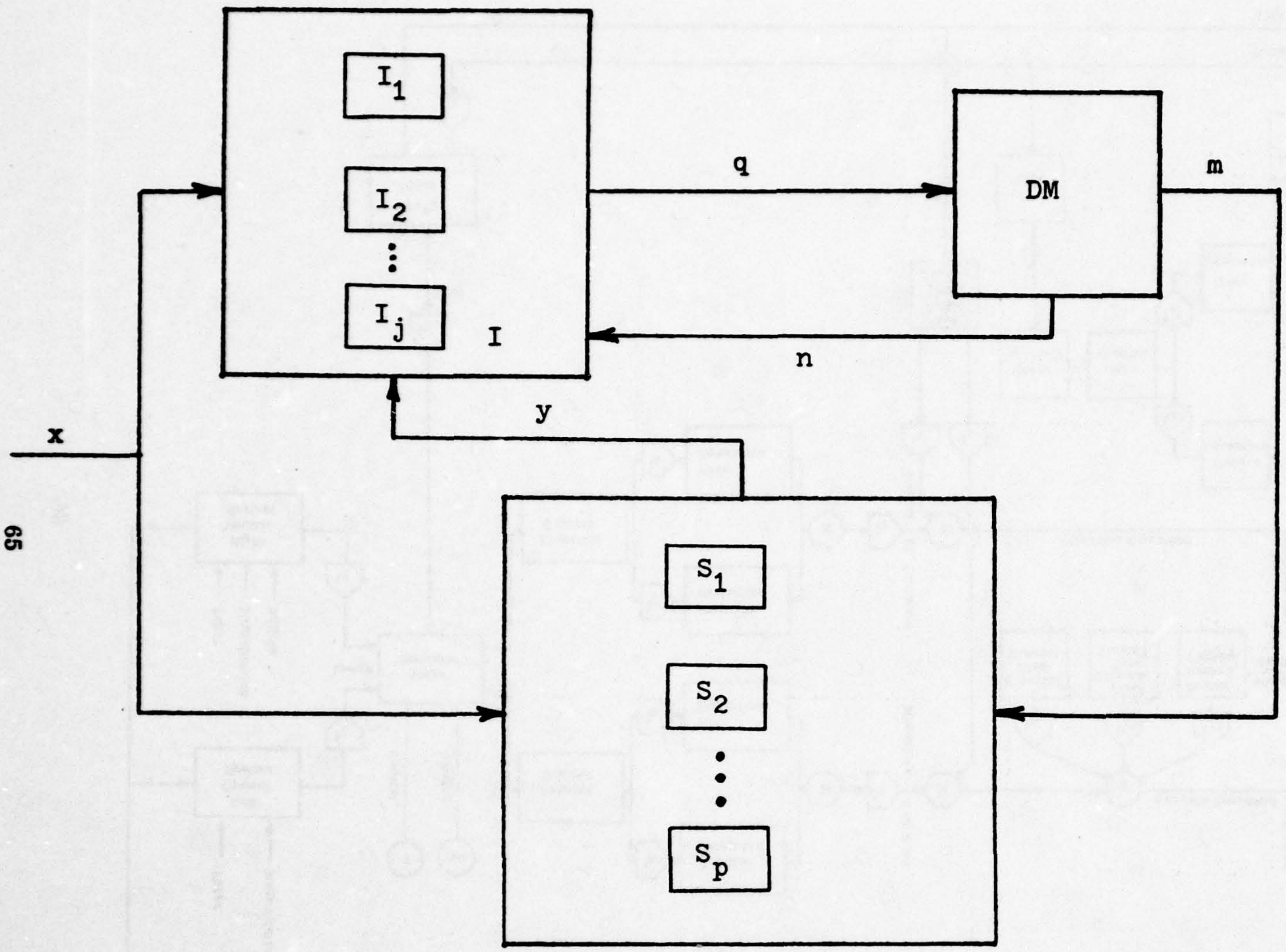


figure 1

Structure of the System

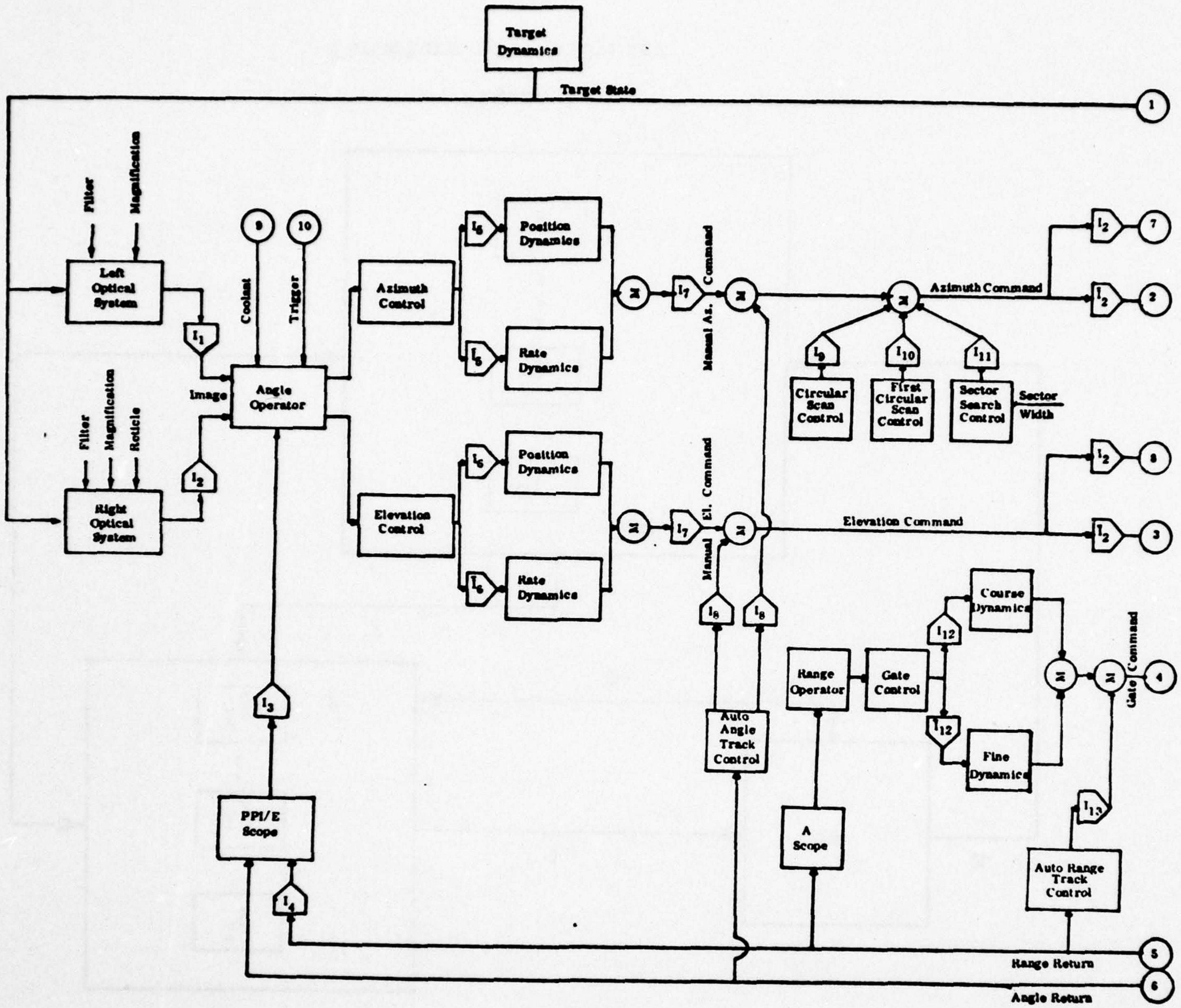


Figure 2A. --Tracking System

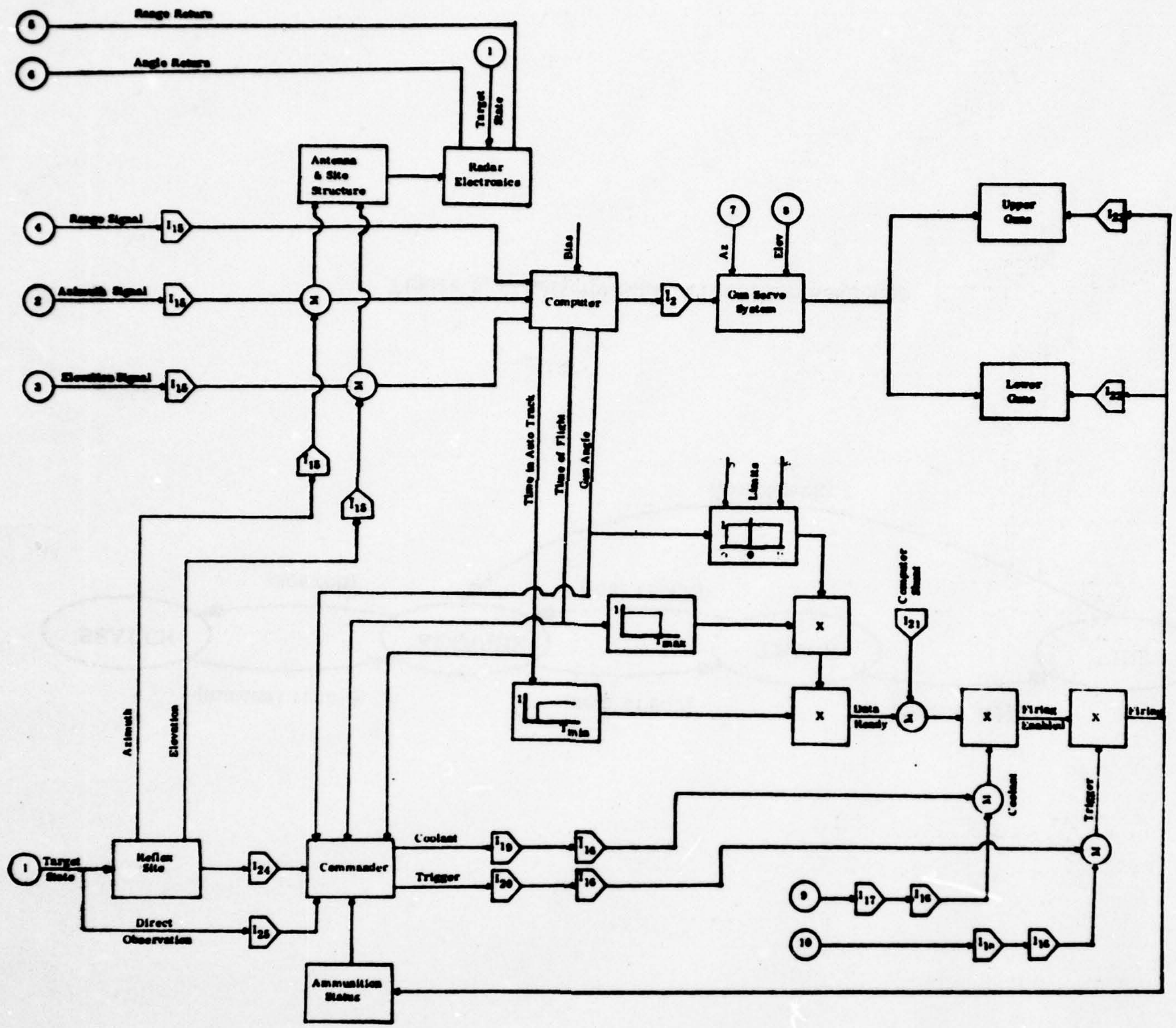


Figure 2B.--Commander System

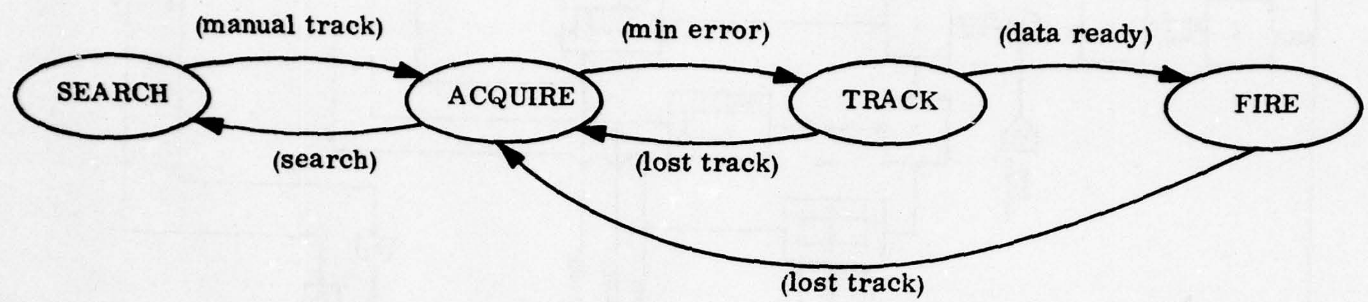


Figure 3. --Activity State Transition Sequence

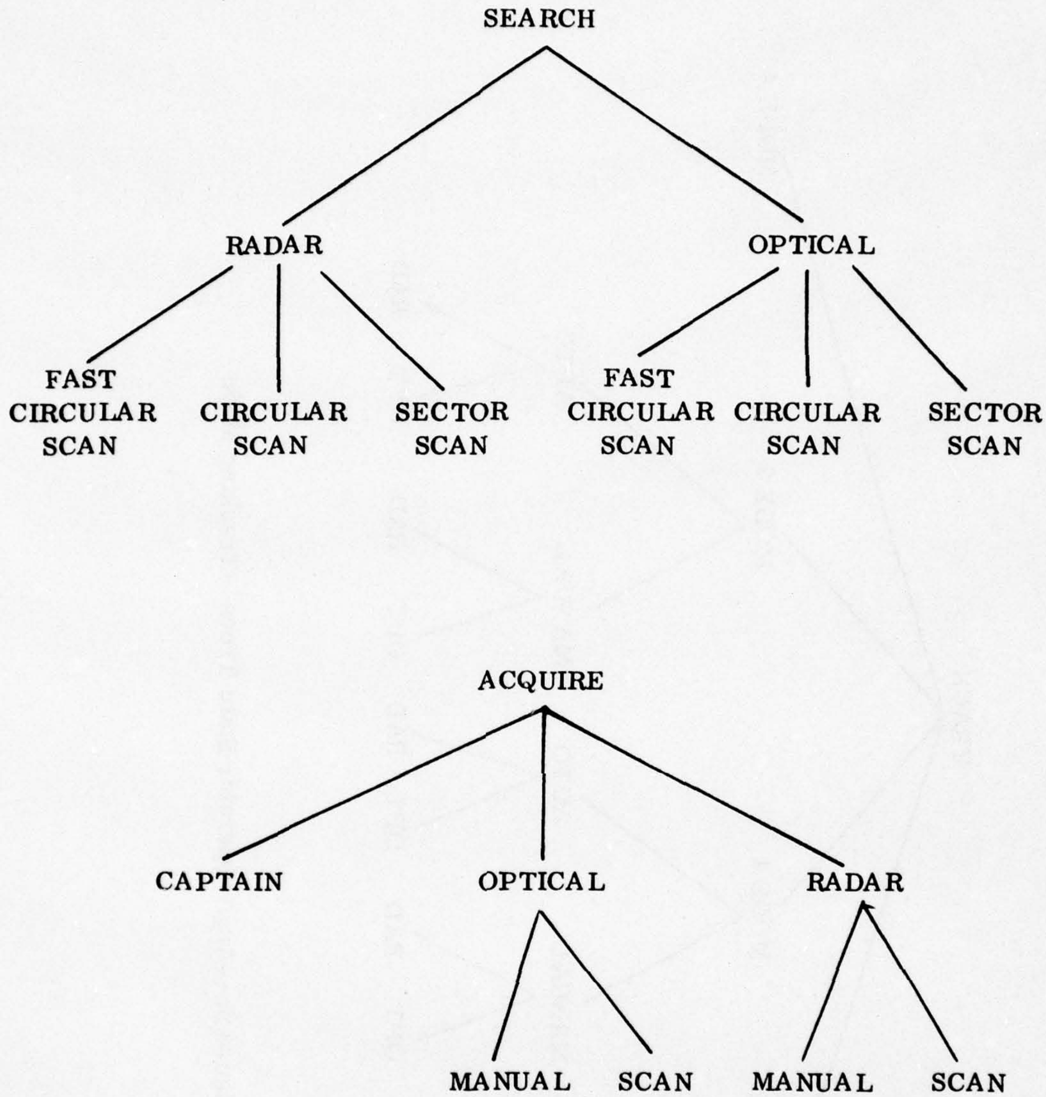


Figure 4. --Angle Operator State Trees-- Search and Acquire Modes

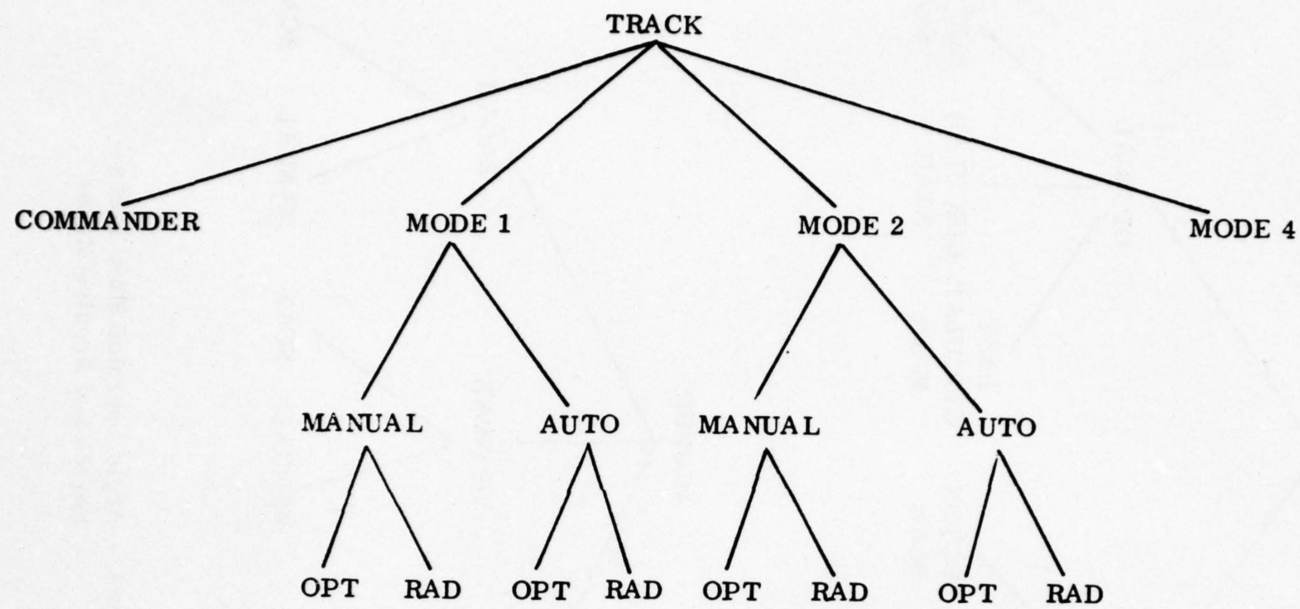


Figure 5.--Angle Operator State Trees--Tracking Mode

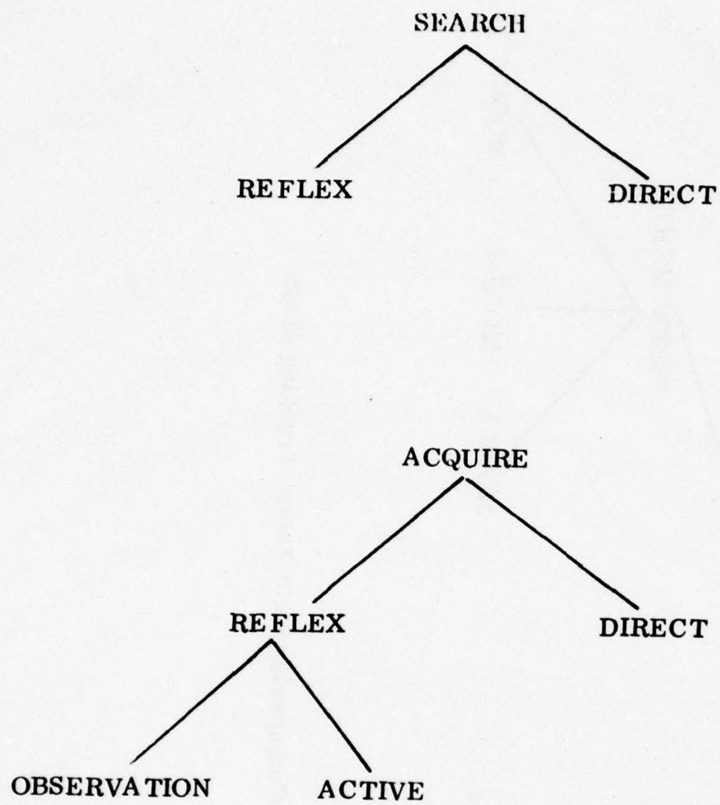


Figure 6.--Commander State Trees--
Search and Acquire Modes

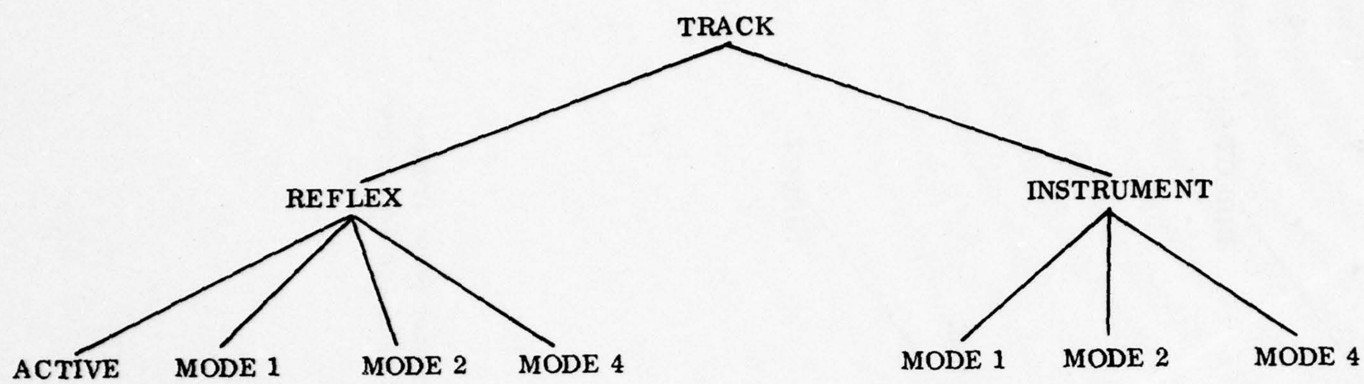
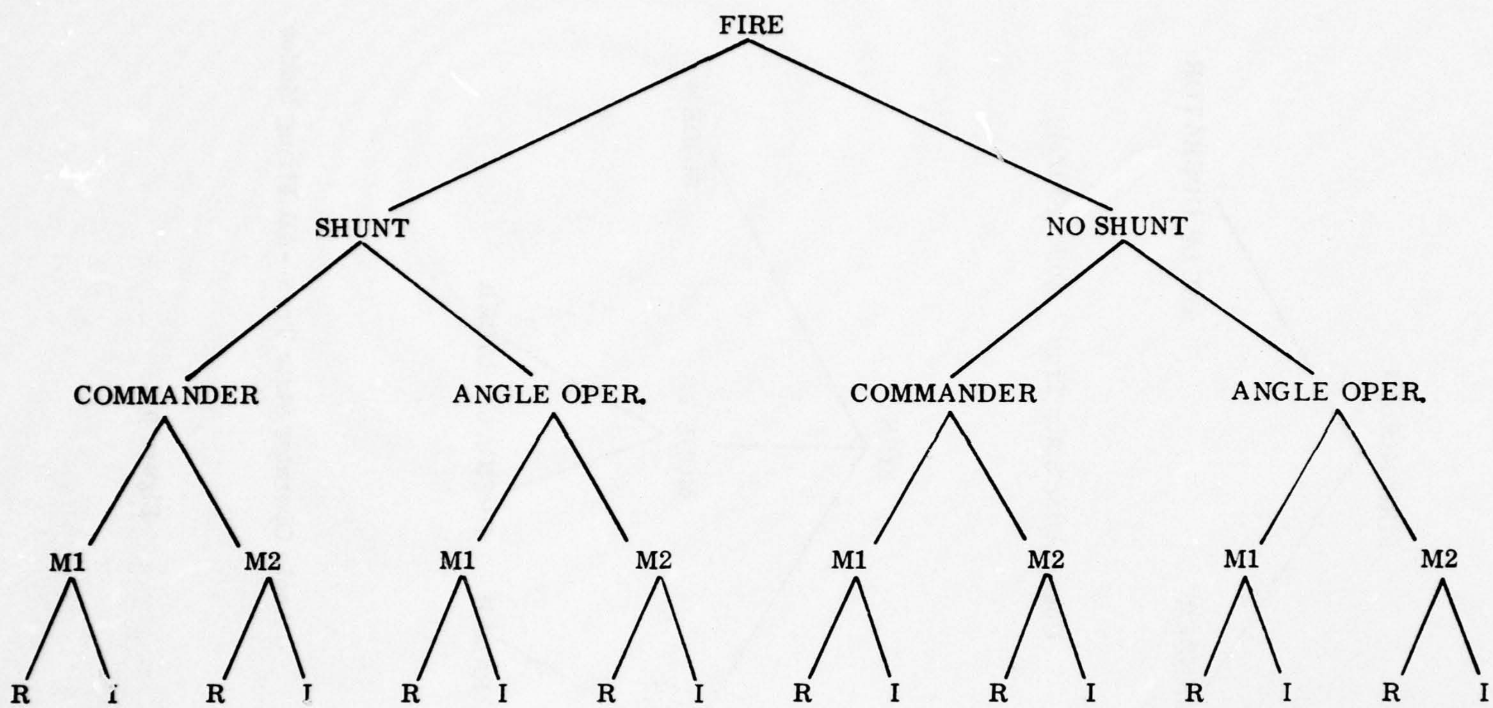
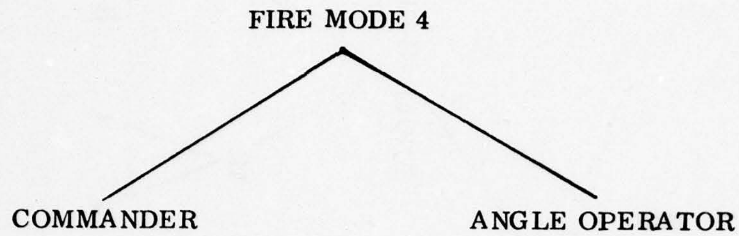


Figure 7. --Commander State Tree--Tracking Mode

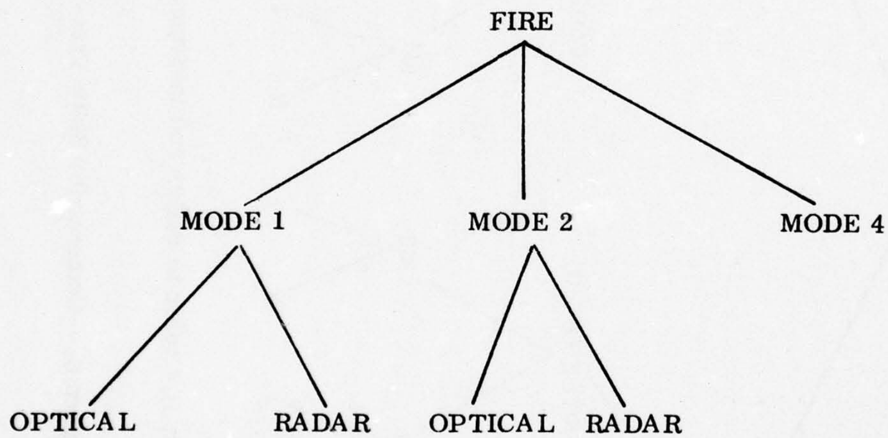


M1, M2 refer to modes 1 and 2; R, I refer to reflex and instrument

Figure 8. --Commander State Tree--Fire Mode



Commander State Tree - Mode 4 Firing



Angle Operator State Tree--All Firing Modes

Figure 9

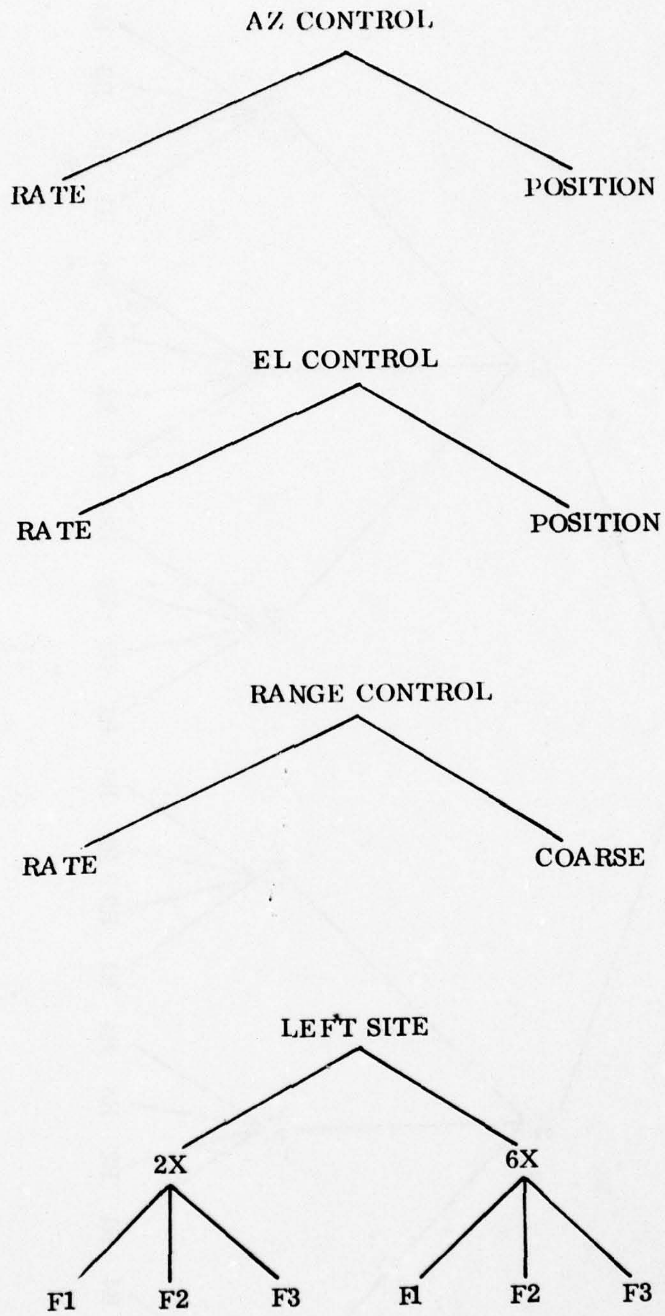


Figure 10. --Hardware State Trees

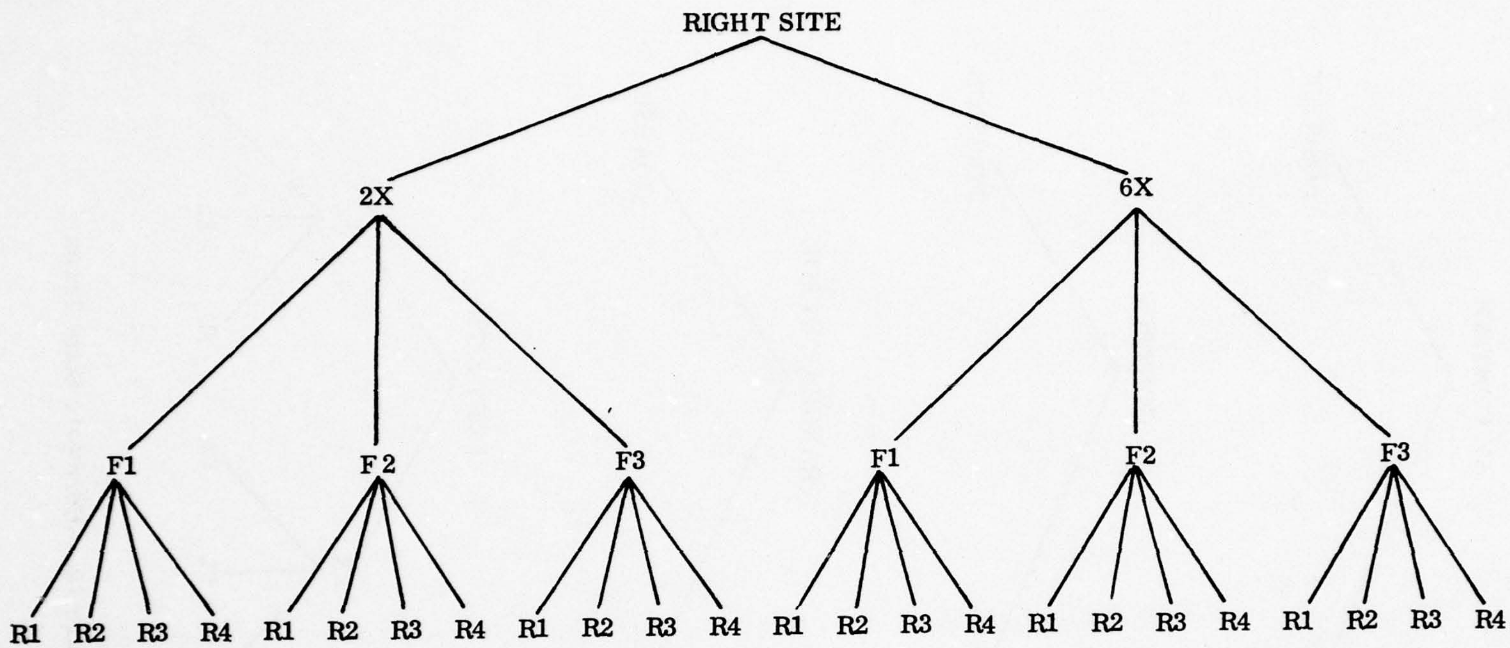


Figure 11. --Right Site State Tree

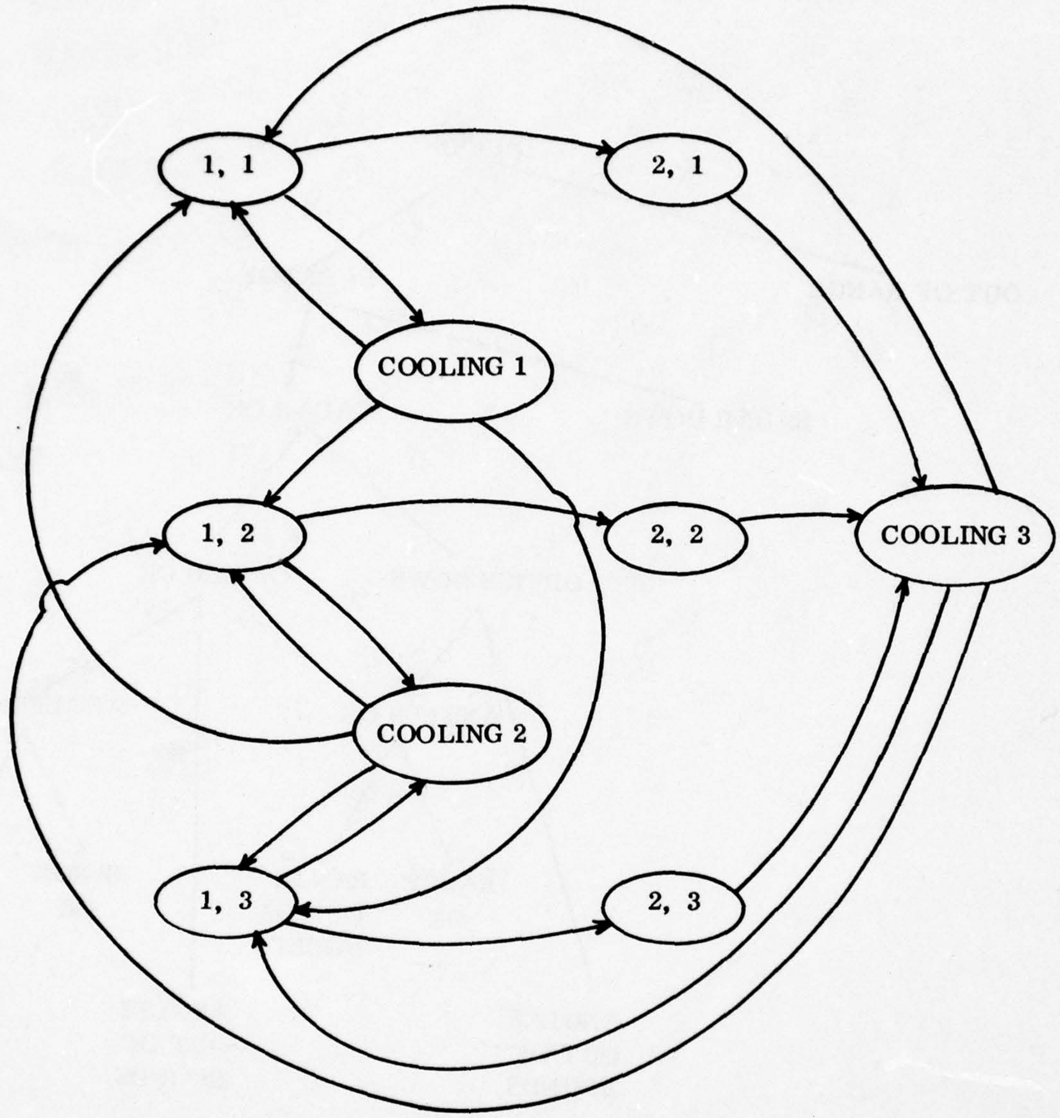


Figure 12. --Firing Doctrine States

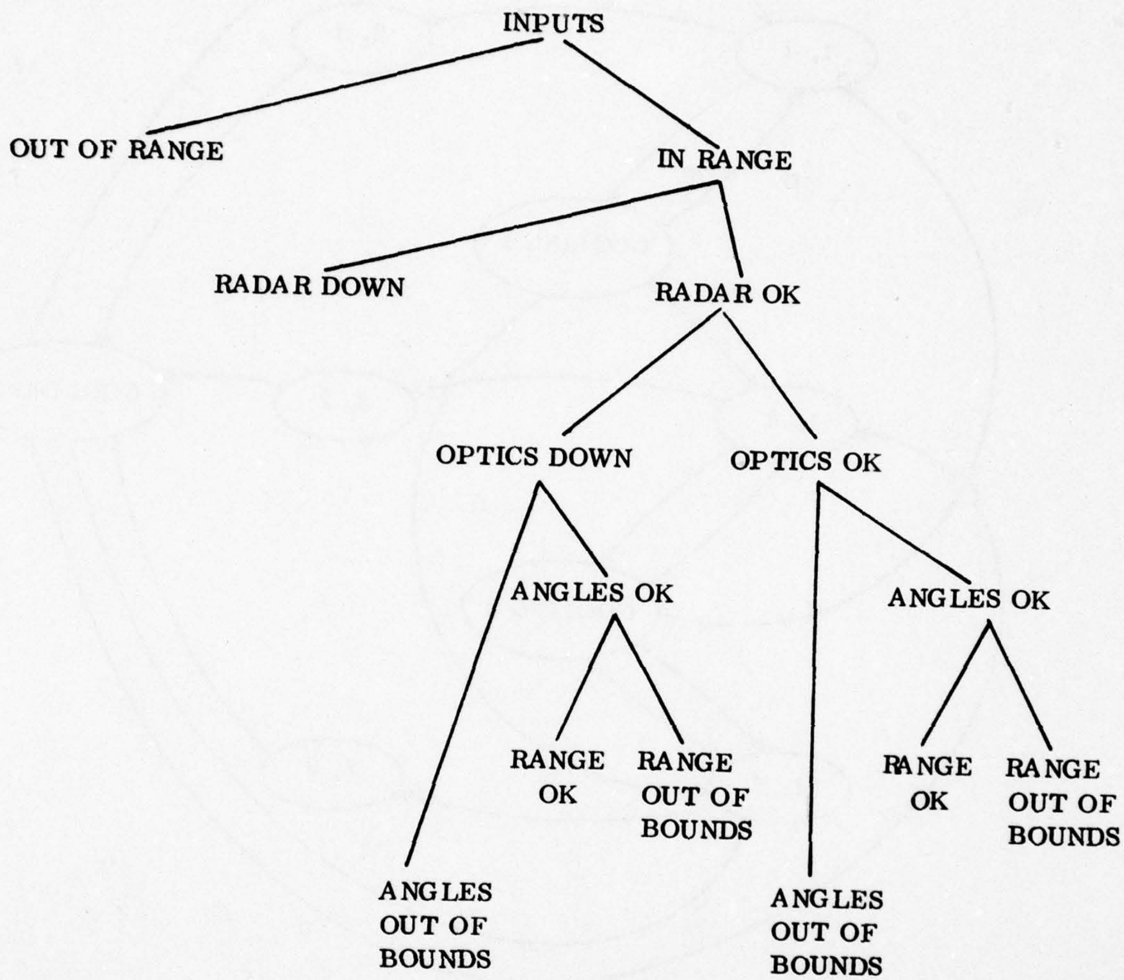


Figure 13. --Input Determinators

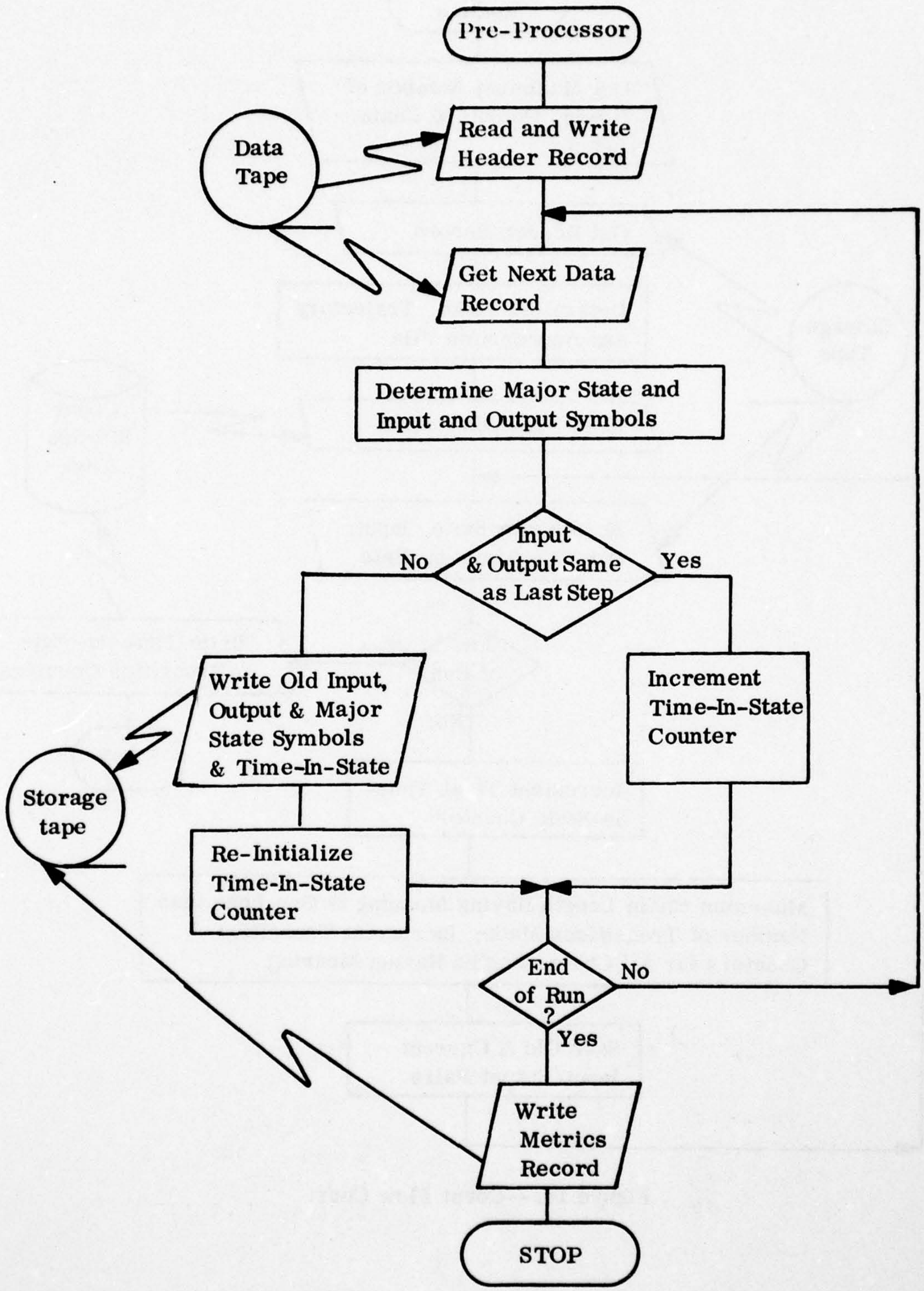


Figure 14. -- Pre-Processing Flow Chart

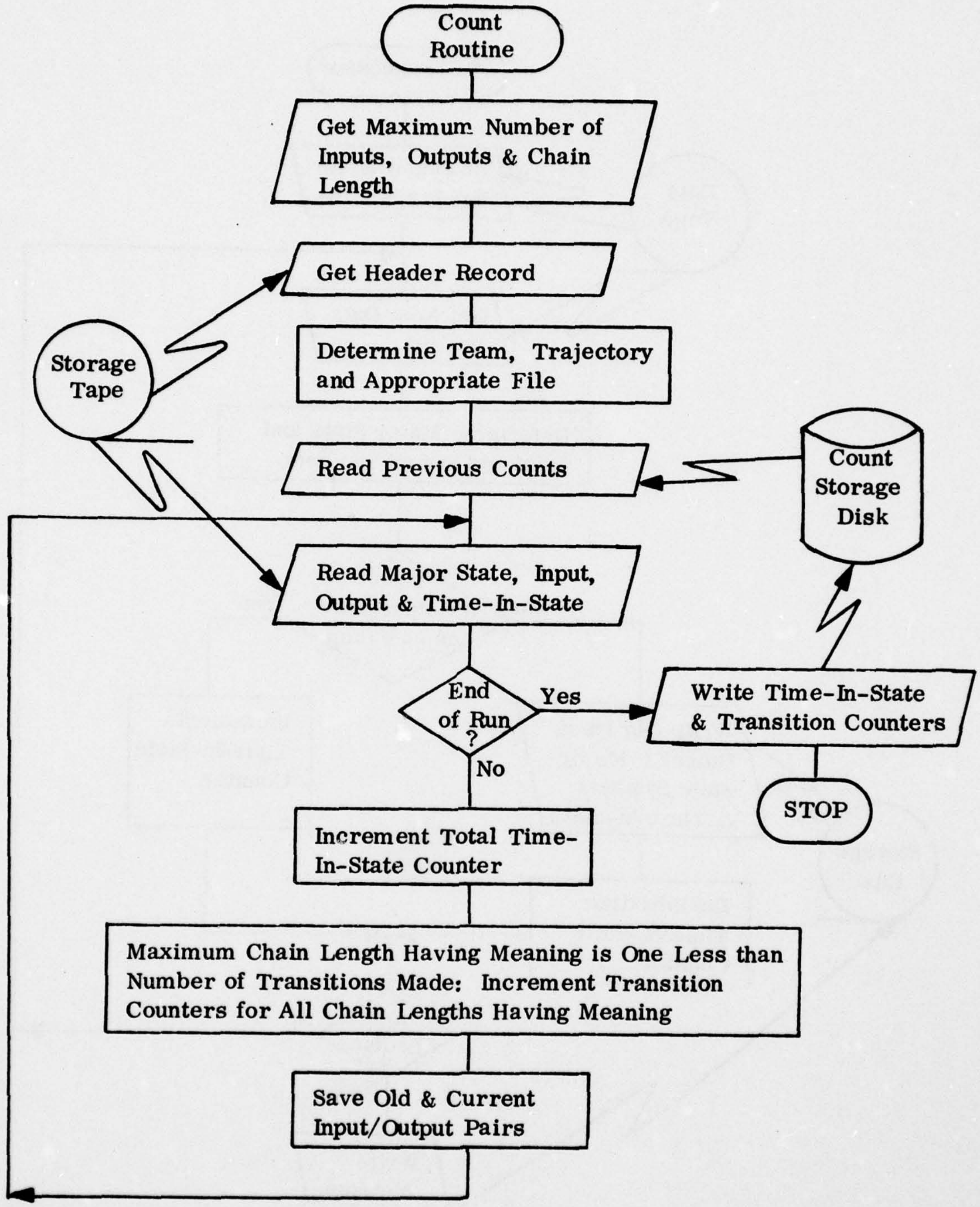


Figure 14.--Count Flow Chart

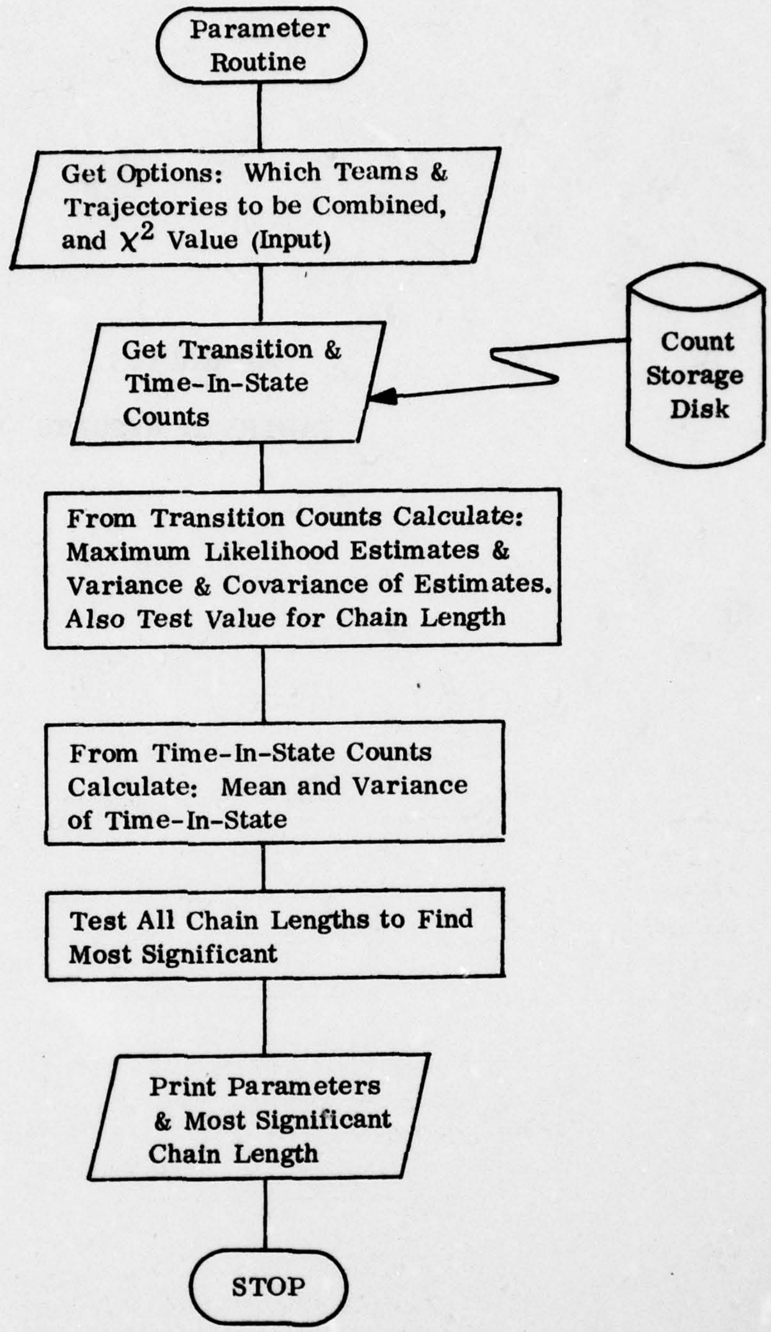


Figure 16.--Parameter Estimation Routine

APPENDIX 7

TABLES OF RESULTS

Table 6

Input Definitions

Input Number	Definition
1	Target has not appeared or range $\geq$ 20,000 ft.
2	Optics and radar blanked
3	Radar blanked and az/el errors out of bounds*
4	Radar blanked and az/el and range errors in bounds
5	Radar blanked, az/el errors in bounds, range errors out of bounds
6	Radar not blanked, az/el errors out of bounds
7	Radar not blanked, az/el and range errors in bounds
8	Radar not blanked, az/el errors in bounds, range out of bounds

* Bounds for errors:

$|\text{az error}| \leq 1120$ units $= 6.3^\circ$

$|\text{el error}| \leq 80$ units $= 2.0^\circ$

$|\text{range error}| \leq 76$ units $= 2432$ feet

Table 7

Operational Definitions of Major States

Major State Number	Descriptor	Enabling Condition
1	Search	Circular or sector search switches set
2	Acquire	Manual control with az/el <u>or</u> range errors out of bounds
3	Track	Auto track switch set <u>or</u> manual control with all errors in bounds

Table 8

Operational Definition of System Activity States

Major State Number	State Number	Description
1	1	Search state, optics in use
1	2	Search state, radar in use
2	1	Acquire state, optics in use
2	2	Acquire state, radar in use
3	1	Track state, mode 1, manual control, radar display
3	2	Track state, mode 1, manual control, optics display
3	3	Track state, mode 1, auto control, radar display
3	4	Track state, mode 1, auto control, optics display
3	5	Track state, mode 2, manual control, radar display
3	6	Track state, mode 2, manual control, optics display
3	7	Track state, mode 2, auto control, radar display
3	8	Track state, mode 2, auto control, optics display

Table 9

Summary of Input and State Occurrences

Input	Number of Occurrences
1	665
2	37
3	388
4	616
5	852
6	975
7	3314
8	2741

Major State	State	Number of Occurrences
1	1	4
1	2	1394
2	1	704
2	2	1290
3	1	526
3	2	514
3	3	4229
3	4	661
3	5	96
3	6	0
3	7	158
3	8	12

Table 10

Number of State Occupancies and Transitions

All Teams/All Trajectories

Major State	Input	State	Number times state occupied	Number transitions from state
1	1	1	2	2
1	1	2	508	2
1	2	1	0	0
1	2	2	0	0
1	3	1	0	0
1	3	2	0	0
1	4	1	0	0
1	4	2	0	0
1	5	1	0	0
1	5	2	0	0
1	6	1	2	1
1	6	2	482	2
1	7	1	0	0
1	7	2	181	0
1	8	1	0	0
1	8	2	221	0
2	1	1	0	0
2	1	2	114	0
2	2	1	11	4
2	2	2	5	3
2	3	1	171	7
2	3	2	191	117
2	4	1	0	0
2	4	2	0	0
2	5	1	242	9
2	5	2	292	72
2	6	1	56	1
2	6	2	338	18
2	7	1	0	0
2	7	2	0	0
2	8	1	224	9
2	8	2	350	24
3	1	1	21	0
3	1	2	0	0
3	1	3	0	0
3	1	4	0	0
3	1	5	20	4

AD-A053 017 OHIO STATE UNIV RESEARCH FOUNDATION COLUMBUS F/G 5/8
IDENTIFICATION OF FINITE STATE MODELS OF A HUMAN OPERATOR.(U)
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UNCLASSIFIED 760518 (MPN 784556) AFOSR-TR-78-0617 NL

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Table 10 (continued)

Major State	Input	State	Number times state occupied	Number transitions from state
3	1	6	0	0
3	1	7	0	0
3	1	8	0	0
3	2	1	0	0
3	2	2	7	2
3	2	3	6	0
3	2	4	8	7
3	2	5	0	0
3	2	6	0	0
3	2	7	0	0
3	2	8	0	0
3	3	1	0	0
3	3	2	0	0
3	3	3	5	0
3	3	4	15	0
3	3	5	0	0
3	3	6	0	0
3	3	7	3	1
3	3	8	1	1
3	4	1	168	57
3	4	2	240	30
3	4	3	167	91
3	4	4	40	21
3	4	5	0	0
3	4	6	0	0
3	4	7	1	0
3	4	8	0	0
3	5	1	0	0
3	5	2	0	0
3	5	3	285	28
3	5	4	33	4
3	5	5	0	0
3	5	6	0	0
3	5	7	0	0
3	5	8	0	0
3	6	1	0	0
3	6	2	0	0
3	6	3	64	5
3	6	4	15	1
3	6	5	0	0
3	6	6	0	0
3	6	7	16	7
3	6	8	2	0

Table 10 (Continued)

Major State	Input	State	Number times state occupied	Number transitions from state
3	7	1	337	267
3	7	2	267	211
3	7	3	1982	111
3	7	4	368	314
3	7	5	76	56
3	7	6	0	0
3	7	7	99	61
3	7	8	4	0
3	8	1	0	0
3	8	2	0	0
3	8	3	1720	84
3	8	4	182	129
3	8	5	0	0
3	8	6	0	0
3	8	7	39	26
3	8	8	5	1

Table 11

Transition Probabilities

Aggregated Data - All Teams, All Trajectories

	Number of Occupancies	Number of Transitions
$P(2, 2)^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	11 5	4 3
$P(2, 3) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	171 191	7 117
$P(2, 5) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	242 292	9 72
$P(2, 6) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	56 338	1 18
$P(2, 8) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	224 350	9 24

*P(i, j) denotes transition probabilities for major state i, input j.

Table 11 (Continued)

$$\begin{array}{lcc} & \text{Number of Occupancies} & \text{Number of Transitions} \end{array}$$

$$P(3,1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{c} 21 \\ 0 \\ 0 \\ 0 \\ 20 \\ 0 \\ 0 \\ 0 \end{array} \qquad \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \end{array}$$

$$P(3,2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .14 & .86 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{c} 0 \\ 7 \\ 6 \\ 8 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \qquad \begin{array}{c} 0 \\ 2 \\ 0 \\ 7 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

$$P(3,3) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{array}{c} 0 \\ 0 \\ 5 \\ 15 \\ 0 \\ 0 \\ 3 \\ 1 \end{array} \qquad \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{array}$$

$$P(3,4) = \begin{bmatrix} 0 & .84 & .16 & 0 & 0 & 0 & 0 & 0 \\ .67 & 0 & 0 & .33 & 0 & 0 & 0 & 0 \\ .77 & 0 & 0 & .23 & 0 & 0 & 0 & 0 \\ 0 & .76 & .24 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{c} 168 \\ 240 \\ 167 \\ 40 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} \qquad \begin{array}{c} 57 \\ 30 \\ 91 \\ 21 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

Table 11 (Continued)

$$P(3,5) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{cc} \text{Number of Occupancies} & \text{Number of Transitions} \\ 0 & 0 \\ 0 & 0 \\ 285 & 28 \\ 33 & 4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$$

$$P(3,6) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .86 & 0 & 0 & 0 & 0 & .14 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 64 & 5 \\ 15 & 1 \\ 0 & 0 \\ 0 & 0 \\ 16 & 7 \\ 2 & 0 \end{array}$$

$$P(3,7) = \begin{bmatrix} 0 & .03 & .97 & 0 & 0 & 0 & 0 & 0 \\ .04 & 0 & 0 & .96 & 0 & 0 & 0 & 0 \\ .09 & 0 & 0 & .91 & 0 & 0 & 0 & 0 \\ 0 & .003 & .997 & 0 & 0 & 0 & 0 & 0 \\ .04 & 0 & 0 & 0 & 0 & 0 & .96 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .98 & 0 & 0 & 0 & 0 & .02 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{cc} 337 & 267 \\ 267 & 211 \\ 1982 & 111 \\ 368 & 314 \\ 76 & 56 \\ 0 & 0 \\ 99 & 61 \\ 4 & 0 \end{array}$$

$$P(3,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 1720 & 84 \\ 182 & 129 \\ 0 & 0 \\ 0 & 0 \\ 39 & 26 \\ 5 & 1 \end{array}$$

Table 12

Time in State Summary

Major State	Input	State	Sample Size	Average Time (sec)	Variance (sec^2)
1	1	1	2	1.10	.02
1	1	2	2	1.10	.02
1	6	1	1	6.37	--
1	6	2	1	6.37	--
2	2	1	4	.71	.11
2	2	2	3	.69	.15
2	3	1	7	.78	.49
2	3	2	117	.63	.16
2	5	1	9	.86	.37
2	5	2	72	.70	.40
2	6	1	1	.63	--
2	6	2	18	.59	.14
2	8	1	9	1.10	1.16
2	8	2	24	1.03	.79
3	1	5	4	.81	.22
3	2	2	2	.13	.01
3	2	4	7	.58	.10
3	3	7	1	3.63	--
3	3	8	1	3.63	--
3	4	1	57	1.42	2.32
3	4	2	30	1.55	1.94
3	4	3	91	.70	.34
3	4	4	21	1.20	.75
3	5	3	28	1.06	.34
3	5	4	4	.63	..16
3	6	3	5	.88	.40
3	6	4	1	.37	--
3	6	7	7	1.45	4.54
3	7	1	267	.46	.34
3	7	2	211	.62	.45
3	7	3	111	2.53	23.81
3	7	4	314	.96	3.57
3	7	5	56	.34	.12
3	7	7	61	.65	.45
3	8	3	84	3.57	16.48
3	8	4	129	1.48	2.82
3	8	7	26	.52	.25
3	8	8	1	.73	--

Table 13

Transition Probabilities and Time in State with Inputs 7 and 8 Combined

$$P(3,7) = \begin{bmatrix} 0 & .03 & .97 & 0 & 0 & 0 & 0 & 0 \\ .04 & 0 & 0 & .96 & 0 & 0 & 0 & 0 \\ .05 & 0 & 0 & .95 & 0 & 0 & 0 & 0 \\ 0 & .002 & .998 & 0 & 0 & 0 & 0 & 0 \\ .04 & 0 & 0 & 0 & 0 & 0 & .96 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .99 & 0 & 0 & 0 & 0 & .01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

State	Average Time	Variance
1	.46	.77
2	.62	1.21
3	2.98	38.62
4	1.11	5.88
5	.34	.34
6	0	0
7	.61	1.13
8	.73	1.08

Table 14

Team 1 - All Trajectories

$$P(3,4) = \begin{bmatrix} 0 & .71 & .29 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .64 & 0 & 0 & .36 & 0 & 0 & 0 & 0 \\ 0 & .5 & .5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of Occupancies	Number of Transitions
30	7
75	3
29	14
11	2
0	0
0	0
0	0
0	0

$$P(3,7) = \begin{bmatrix} 0 & .04 & .96 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ .10 & 0 & 0 & .9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ .03 & 0 & 0 & 0 & 0 & 0 & .97 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of Occupancies	Number of Transitions
64	46
93	62
457	41
128	92
44	30
0	0
57	31
0	0

$$P(3,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of Occupancies	Number of Transitions
0	0
0	0
411	36
83	55
0	0
0	0
26	19
0	0

Table 15

Team 2 - All Trajectories

Number of Occupancies | Number of Transitions

$$P(3,4) = \begin{bmatrix} 0 & .73 & .27 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ .60 & 0 & 0 & .40 & 0 & 0 & 0 & 0 \\ 0 & .94 & .06 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 48 & 22 \\ 82 & 3 \\ 70 & 40 \\ 23 & 16 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 00 & 0 \end{array}$$

$$P(3,7) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & .01 & .99 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 69 & 51 \\ 70 & 59 \\ 404 & 19 \\ 92 & 84 \\ 4 & 3 \\ 0 & 0 \\ 3 & 2 \\ & \end{array}$$

$$P(3,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 317 & 10 \\ 32 & 23 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$$

Table 16

Team 3 - All Trajectories

$$
\begin{array}{lcc}
 & \text{Number of Occupancies} & \text{Number of Transitions} \\
P(3,4) = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
.89 & 0 & 0 & .11 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
& \begin{matrix} 62 \\ 61 \\ 32 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}
& \begin{matrix} 21 \\ 19 \\ 19 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}
\end{array}
$$

$$
\begin{array}{lcc}
P(3,7) = \begin{bmatrix}
0 & .04 & .96 & 0 & 0 & 0 & 0 & 0 \\
.07 & 0 & 0 & .93 & 0 & 0 & 0 & 0 \\
.11 & 0 & 0 & .89 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
& \begin{matrix} 76 \\ 51 \\ 552 \\ 69 \\ 25 \\ 0 \\ 36 \\ 0 \end{matrix}
& \begin{matrix} 72 \\ 45 \\ 19 \\ 66 \\ 22 \\ 0 \\ 26 \\ 0 \end{matrix}
\end{array}
$$

$$
\begin{array}{lcc}
P(3,8) = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
& \begin{matrix} 0 \\ 0 \\ 489 \\ 29 \\ 0 \\ 0 \\ 12 \\ 0 \end{matrix}
& \begin{matrix} 0 \\ 0 \\ 12 \\ 22 \\ 0 \\ 0 \\ 7 \\ 0 \end{matrix}
\end{array}
$$

Table 17

Team 4 - All Trajectories

$$
\begin{array}{rc}
 & \\
P(3,4) = & \left[\begin{array}{cccccccc}
0 & .86 & .14 & 0 & 0 & 0 & 0 & 0 \\
.60 & 0 & 0 & .40 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
\quad
\begin{array}{cc}
\text{Number of Occupancies} & \text{Number of Transitions} \\
28 & 7 \\
22 & 5 \\
36 & 18 \\
4 & 2 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 0
\end{array}
$$

$$
P(3,7) = \left[\begin{array}{cccccccc}
0 & .03 & .97 & 0 & 0 & 0 & 0 & 0 \\
.13 & 0 & 0 & .87 & 0 & 0 & 0 & 0 \\
.13 & 0 & 0 & .87 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
00 & 0 & .5 & 0 & 0 & 0 & 0 & .5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\quad
\begin{array}{cc}
128 & 98 \\
53 & 45 \\
569 & 32 \\
79 & 72 \\
3 & 1 \\
0 & 0 \\
3 & 2 \\
4 & 0
\end{array}
$$

$$
P(3,8) = \left[\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\quad
\begin{array}{cc}
0 & 0 \\
0 & 0 \\
503 & 26 \\
34 & 29 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
5 & 1
\end{array}
$$

Table 18

Trajectory Number 1 - All Teams

$$P(3,4) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ .67 & 0 & 0 & .33 & 0 & 0 & 0 & 0 \\ .55 & 0 & 0 & .45 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} \text{Number of Occurrances} & \text{Number of Transitions} \\ 13 & 7 \\ 38 & 3 \\ 16 & 11 \\ 8 & 5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$$

$$P(3,7) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ .03 & 0 & 0 & .97 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 29 & 25 \\ 38 & 29 \\ 381 & 22 \\ 60 & 51 \\ 6 & 6 \\ 0 & 0 \\ 13 & 6 \\ 0 & 0 \end{array}$$

$$P(3,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 371 & 13 \\ 30 & 24 \\ 0 & 0 \\ 0 & 0 \\ 5 & 5 \\ 0 & 0 \end{array}$$

Table 19

Trajectory Number 2 - All Teams

$$P(3,4) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & .5 & 0 & .5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} \text{Number of} & \text{Number of} \\ \text{Occupancies} & \text{Transitions} \\ 2 & 1 \\ 18 & 1 \\ 15 & 3 \\ 4 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$$

$$P(3,7) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ .04 & 0 & 0 & .96 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ .17 & 0 & 0 & 0 & 0 & 0 & .83 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 36 & 26 \\ 36 & 27 \\ 271 & 11 \\ 46 & 41 \\ 8 & 6 \\ 0 & 0 \\ 12 & 3 \\ 0 & 0 \end{array}$$

$$P(3,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 205 & 3 \\ 15 & 12 \\ 0 & 0 \\ 0 & 0 \\ 11 & 7 \\ 0 & 0 \end{array}$$

Table 20

Trajectory Number 4 - All Teams

$$P(3,4) = \begin{bmatrix} 0 & .91 & .09 & 0 & 0 & 0 & 0 & 0 \\ .75 & 0 & 0 & .25 & 0 & 0 & 0 & 0 \\ .75 & 0 & 0 & .25 & 0 & 0 & 0 & 0 \\ 0 & .67 & .33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of Occupancies	Number of Transitions
17	11
29	4
15	8
5	3
0	0
0	0
0	0
0	0

$$P(3,7) = \begin{bmatrix} 0 & .11 & .89 & 0 & 0 & 0 & 0 & 0 \\ .04 & 0 & 0 & .96 & 0 & 0 & 0 & 0 \\ .08 & 0 & 0 & .92 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of Occupancies	Number of Transitions
36	28
32	26
222	13
59	41
9	6
0	0
11	8
0	0

$$P(3,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of Occupancies	Number of Transitions
0	0
0	0
189	18
44	23
0	0
0	0
3	3
0	0

Table 21

Trajectory Number 7 - All Teams

$$\begin{array}{lccc}
 & & \begin{matrix}\text{Number of}\\ \text{Occupancies}\end{matrix} & \begin{matrix}\text{Number of}\\ \text{Transitions}\end{matrix} \\
P(3,4) = & \begin{bmatrix} 0 & .83 & .17 & 0 & 0 & 0 & 0 & 0 \\ .78 & 0 & 0 & .22 & 0 & 0 & 0 & 0 \\ .80 & 0 & 0 & .20 & 0 & 0 & 0 & 0 \\ 0 & .50 & .50 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 32 \\ 42 \\ 26 \\ 9 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 6 \\ 9 \\ 20 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\
 & & & \\
P(3,7) = & \begin{bmatrix} 0 & .02 & .98 & 0 & 0 & 0 & 0 & 0 \\ .08 & 0 & 0 & .92 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 52 \\ 43 \\ 282 \\ 55 \\ 20 \\ 0 \\ 18 \\ 0 \end{matrix} & \begin{matrix} 47 \\ 40 \\ 13 \\ 50 \\ 0 \\ 0 \\ 13 \\ 0 \end{matrix} \\
 & & & \\
P(3,8) = & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 232 \\ 17 \\ 0 \\ 0 \\ 2 \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 12 \\ 10 \\ 0 \\ 0 \\ 1 \\ 0 \end{matrix}
\end{array}$$

Table 22

Trajectory Number 8 - All Teams

$$\begin{array}{lccc}
 & & \text{Number of Occupancies} & \text{Number of Transitions} \\
P(3,4) = & \begin{bmatrix} 0 & .78 & .22 & 0 & 0 & 0 & 0 & 0 \\ .33 & 0 & 0 & .67 & 0 & 0 & 0 & 0 \\ .83 & 0 & 0 & .17 & 0 & 0 & 0 & 0 \\ 0 & .50 & 0 & .50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 30 \\ 30 \\ 16 \\ 5 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} & \begin{array}{c} 9 \\ 3 \\ 6 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\
 & & & \\
P(3,7) = & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .93 & 0 & 0 & 0 & 0 & .07 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 67 \\ 22 \\ 354 \\ 36 \\ 19 \\ 0 \\ 23 \\ 4 \end{array} & \begin{array}{c} 51 \\ 14 \\ 14 \\ 27 \\ 13 \\ 0 \\ 15 \\ 0 \end{array} \\
 & & & \\
P(3,8) = & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ 0 \\ 365 \\ 54 \\ 0 \\ 0 \\ 7 \\ 5 \end{array} & \begin{array}{c} 0 \\ 0 \\ 27 \\ 47 \\ 0 \\ 0 \\ 2 \\ 1 \end{array}
\end{array}$$

Table 23

Trajectory Number 9 - All Teams

$$P(3,4) = \begin{bmatrix} 0 & .82 & .18 & 0 & 0 & 0 & 0 & 0 \\ .83 & 0 & 0 & .17 & 0 & 0 & 0 & 0 \\ .76 & 0 & 0 & .24 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} \text{Number of Occupancies} & \text{Number of Transitions} \\ 32 & 11 \\ 33 & 6 \\ 41 & 17 \\ 5 & 4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$$

$$P(3,7) = \begin{bmatrix} 0 & .08 & .92 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ .22 & 0 & 0 & .78 & 0 & 0 & 0 & 0 \\ 0 & .02 & .98 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 48 & 38 \\ 50 & 35 \\ 264 & 18 \\ 58 & 52 \\ 4 & 3 \\ 0 & 0 \\ 10 & 5 \\ 0 & 0 \end{array}$$

$$P(3,8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 264 & 11 \\ 21 & 13 \\ 0 & 0 \\ 0 & 0 \\ 7 & 6 \\ 0 & 0 \end{array}$$

Table 24

Trajectory Number 10 - All Teams

$$
\begin{array}{cc}
 & \begin{array}{cc} \text{Number of Occupancies} & \text{Number of Transitions} \end{array} \\
P(3,4) = \begin{bmatrix}
0 & .75 & .25 & 0 & 0 & 0 & 0 & 0 \\
.5 & 0 & 0 & .5 & 0 & 0 & 0 & 0 \\
.92 & 0 & 0 & .08 & 0 & 0 & 0 & 0 \\
0 & .67 & .33 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} &
\begin{array}{cc}
42 & 12 \\
50 & 4 \\
38 & 26 \\
4 & 3 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}
\end{array}
$$

$$
\begin{array}{cc}
P(3,7) = \begin{bmatrix}
0 & .02 & .98 & 0 & 0 & 0 & 0 & 0 \\
.08 & 0 & 0 & .93 & 0 & 0 & 0 & 0 \\
.25 & 0 & 0 & .75 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
.11 & 0 & 0 & 0 & 0 & 0 & .89 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} &
\begin{array}{cc}
69 & 52 \\
46 & 40 \\
208 & 20 \\
54 & 52 \\
10 & 9 \\
0 & 0 \\
12 & 8 \\
0 & 0
\end{array}
\end{array}
$$

$P(3,8) =$ No data